A MATHEMATICAL EXPRESSION OF STEM FORM AND VOLUME FOR LOBLOLLY PINE IN SOUTHERN BRAZIL

By

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Scope and Method of Study: In the production of quality timber, a more efficient use of wood will be achieved when forest inventory data are collected according to the industry's needs. The investigation of stem form can provide the decision-maker with the necessary information. It is the purpose of this biometric study to describe the development of a mathematical expression of tree stem form or taper, and the transformation of such a model into a volume function. Data for this study were obtained in loblolly pine plantations in southern Brazil. Hohenadl's method of stem volume estimation was used in collecting diameter data of 188 stems. A multivariate exploratory technique, principal component analysis (PCA), was used in the taper model building process. That statistical technique required the solution to the eigenequation of the sum of squares and cross-products matrix prepared from the data. Seven eigenvalues and associated eigenvectors were obtained.

Findings and Conclusions: The objective of this study was successfully achieved using principal components analysis. The first eigenvalue accounted for 99.72 percent of the stem form variability, and its associated eigenvector was assumed to be an expression of average stem form. A taper model was developed and after integration provided a volume function. Volume estimates for all 188 sampled stems were obtained by using both procedures: the taper-derived volume function, and Hohenadl's method. Residuals of the taper-derived volume function estimates over those of Hohenadl's method were computed. Percent residuals ranged from -30.60 to 28.50 and were uniformly distributed over most of the data range. Principal component analysis was found to be an extremely useful technique for the investigation of stem form.

ADVISER'S APPROVAL ______________________________

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This study is dedicated to Luis Roberto Ahrens, my son, whose existence has always been, above any other reason, the main inspiration for my life.

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CHAPTER I

THE RESEARCH PROBLEM

Introduction

Following the world population growth as well as the changing living standards of society, demand for natural resources is constantly increasing. This naturally reflects on higher values for forest products and consequently for wood. In addition, modern manufacturing methods require more precise and detailed information on raw material supply.

In the context of multi-product intensive management of pine plantations in southern Brazil, several questions have been asked regarding the comparative efficiency of different stand density, thinning and pruning methods and choice of rotation length. Currently however, additional and important questions are being studied with respect to alternative methodologies for the estimation of important parameters that will quantify and describe the raw material being produced according to different end-uses.

Dealing with even-aged industrial forests and because of their production oriented management structures, forest inventory data of timberland in southern Brazil must now be collected by more sophisticated procedures. These are necessary to provide the forest manager with more accurate estimates of the wood available by detailed size classes. These measurements are also essential to the decision-maker with respect to policy formation and planning of forest operations directly related to the industry's needs. The information allows managers to make a more comprehensive analysis and consequently to suggest a more efficient use of wood.

Wood volume of a stand is the sum of the volume of each individual tree included in that stand. The volume of a tree has traditionally been defined by height, diameter at breast height and form, and can be estimated through several alternative ways, i.e. water displacement, graphical methods, the use of formulas such as Smalian's, Huber's and Newton's, or the integration of a taper model.

It is the objective of this paper to describe the development of a mathematical expression of tree form or taper, and the transformation of such a model into a tree volume function.

Data for this study were obtained in a loblolly pine plantation located in southern Brazil. This is an introduced conifer of increasing importance for the regional forestry activities of that country.[1]

Justification

Some of the fundamental reasons for this research project are implied in the introduction and statement of the objective. This author believes however, that some other assumptions must be properly explained.

The shape and conceptual sectioning of the stem of a conifer is illustrated with Figure 1. The stems and stumps of trees closely resemble certain geometrical solids (Hush et al., 1972) and their cubic volume may be estimated by applying formulas either before or after they are

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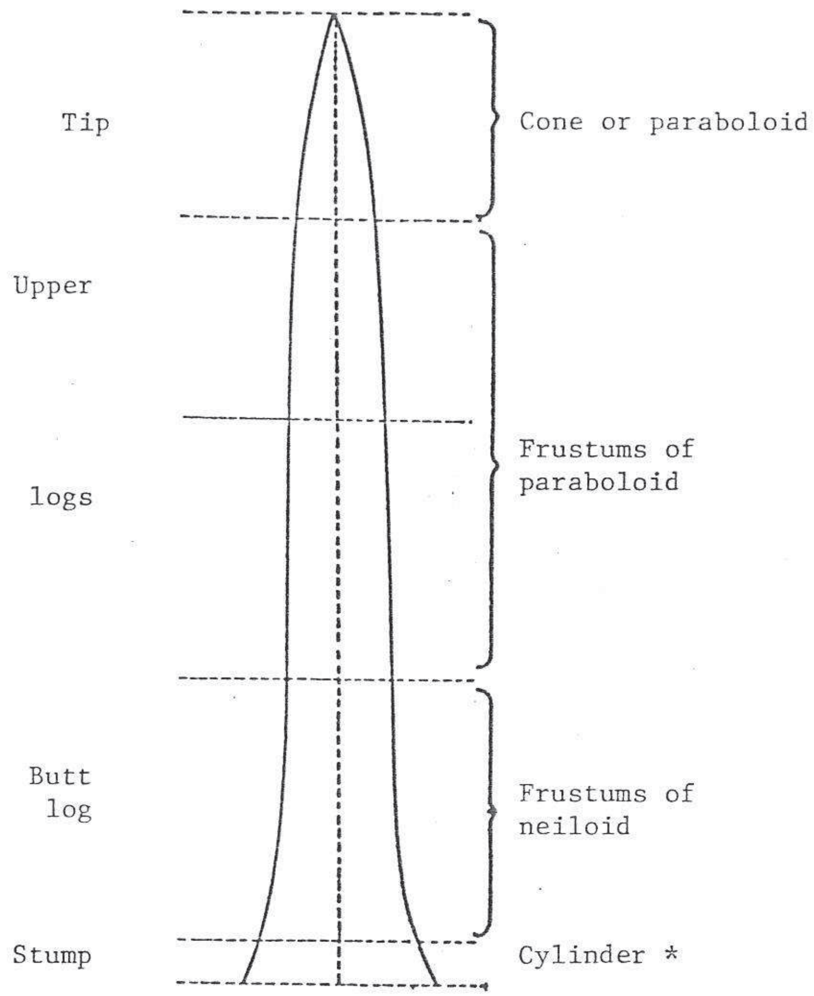
[1]All species cited in this paper are listed in Appendix A with their common and scientific names.



*the stump is usually treated as a cylinder, but has the geometric form of a frustum of neiloid.
Source: Husch et al., 1972, p. 121.

Figure 1. Geometric Forms Frequently Observed in Portions of Tree Stems

felled. However, accurate volume estimation of standing trees has traditionally been demonstrated to be one of the most difficult tasks in forest mensuration. Usually what has been suggested is to measure several diameters along the stem of felled trees and compute an estimate of the volume through the use of any one of the accepted formulas, such as those of Smalian, Huber or Newton. Later, a regression analysis is run for volume prediction purposes having diameter at breast height ($d_{1.30}$), and height (h), as the independent variables.[2]

For some industrial activities the information on the volume of bulk wood is sufficient. In the production of quality timber however, volume must be given according to some merchantability limits, such as logs with some given diameters or lengths. This type of data can certainly be produced with the traditional methods. However the prediction ability is always conditioned by the way the field data was collected for the analysis. Investigators and industry recognize the need for a more flexible system.

With the availability of high-speed data processing equipment, the study of another approach to the problem is possible. That is the direct investigation of the form of tree stems. A taper model is a mathematical description of the longitudinal stem profile and, under the assumption that the cross-section is circular at all points along the stem, its volume can be determined by integration. Once the taper model is defined, volume can be obtained for any section of the stem.

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[2]In this paper special emphasis was made on the use of symbols recommended by the International Union of Forestry Research Organizations--IUFRO. All the measurements and computations were done with the metric system.

CHAPTER II

LITERATURE REVIEW

Considerable work has been done on the factors involved in volume estimation because of their importance and complexity. Several investigators have been studying the relationship between stem form and volume by means of searching for an adequate taper function. The objective of any taper study is to quantify the relationship between stem form and some easy-to-measure tree characteristics such as diameter at breast height and tree total height. That relationship must be expressed with as much simplicity as possible.

The historical development of the most significant contributions to this field can be organized into three different major groups of tree taper studies: the early stage of oversimplified models, the use of polynomials and the recently developed use of principal component analysis, PCA. An appreciation of taper and volume systems is also included in this review.

The Early Oversimplified Models

In studying tree stem form, the concept of its expression with a mathematical function was first generated in Sweeden by Höjer (1903) (cited from Behre, 1923). Working with Norway spruce, he was the first investigator to suggest a model for the determination of the diameter of a tree at any point along its stem. The equation he suggested has the form:

$$\frac{d}{d_{4.5}} = b_1 \log \frac{b_2 + \ell/h}{b_2} \qquad (2.1)$$

where d is the estimated diameter at distance ℓ from the tip of a stem with total height h, $d_{4.5}$ is the diameter at the breast height (dbh, measured at 4.5 feet from the ground), and b_1 and b_2 are coefficients to be determined.

Jonson (1910, 1911) reported that Höjer's model was adequate for spruce stands growing from local seed sources. However, diameters were overestimated in the upper sections of stems obtained from compartments established with imported seeds. Jonson added a "biological constant" to Höjer's model and produced a taper function and cubic volume tables for Scots pine.

$$\frac{d}{d_{4.5}} = b_1 \log \frac{b_2 + \ell/(h - b_3)}{b_2} \qquad (2.2)$$

This transformed model however, will not produce any diameter along a portion of the tip of the stem equivalent to b_3. Hence an inconsistency was in fact introduced (cited from Demaerschalk, 1971a).

Following the lead of the European investigators and working with ponderosa pine, Behre (1923, 1927, 1935) suggested a new model that has the form of the ordinary hyperbola:

$$\frac{d}{d_{4.5}} = \frac{\ell/h}{b_1 + b_2(\ell/h)} \qquad (2.3)$$

In this function, d, $d_{4.5}$, ℓ and h have the same meaning as in Höjer's model and, b_1 and b_2 are coefficients obtained by regression analysis. Under the assumption that the form of the stem of conifers can be expressed with a single equation applicable to all species, diameters and heights, Behre's model became the basis for the board-foot volume tables prepared by Mesavage and Girard (1946) and Girard and Bruce (1947, 1949).

According to Bruce and Schumacher (1950), Behre's equation gives a satisfactory result for several species but it fails to fit that portion of the tree stem where the butt swell begins. Correction factors were suggested by the authors of those earlier tables and they were used as a standard reference by many forest operators. Residuals of predicted volumes were considered not significant in practice when the tables were used with native species of western United States.

Atterbury (1973) reported the work done by Wicoff in 1966 to estimate cubic volume of wood on Crown Zellerbach timberland in Northwestern United States. In those studies Wicoff claimed that if the convex hyperboloid described with Behre's equation fits the stem above the form point (i.e. the inflection point as defined by that author), then a concave hyperboloid should be adequate for the portion of the stem below that level. The results obtained with this approach were considered adequate for the objectives of the company at that time. Recently (Liu, 1973), the same company provided loblolly pine data for a principal component analysis.[1] This is an indication that currently a more flexible and reliable mathematical expression of taper is needed.

The main difficulties of the practical use of the equations suggested by Höjer, Jonson and Behre were summarized by Bruce, Curtis and Vancoevering (1968). The computational work involved was excessive and

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[1]Since that was also the technique used in this study, PCA is more completely described in Chapter III, Materials and Methods.

the resulting oversimplified equations did not satisfactorily describe the butt swell and tip.

Among the objectives of those early studies to describe stem form, was also the attempted development of a universal taper function. This author believes it will be hard to accomplish such an objective with a simple model because of the several sources of variability that may influence tree stem form: different species, past management practices, age and growth rates among others. In regard to the observed adjustment of those functions, it must be said that only the inclusion of more coefficients to a basic model, following an empirical method or iterative process, will not necessarily improve its diameter prediction ability. It may even introduce a more serious bias. At that stage the natural extension of taper model building was obviously towards the addition of more independent variables by using transformations of diameter at breast height and tree total height. This necessarily includes the use of polynomial functions to investigate stem form.

The Use of Polynomials

In reviewing the previous work done on stem form, this author observed that the literature contains an extremely significant amount of taper studies using polynomial regression analysis. This reflects what was considered by several investigators a reasonable approach to the problem. A review follows:

To describe stem form above breast height of loblolly pine, Matte (1949) suggested the model

$$\frac{d^2}{d_{4.5}^2} = b_o \frac{h_d^2}{h^2} + b_1 \frac{h_d^3}{h^3} + b_2 \frac{h_d^4}{h^4} \qquad (2.4)$$

where d is the diameter in inches at height h_d from the ground in feet. The dbh outside bark ($d_{4.5}$) is measured in inches at 4.5 feet from the ground. Tree total height (h) is measured in feet.

Osumi (1959), cited by Demaerschalk (1971a), reported a similar model to the one suggested by Matte. Using the same symbols, the form of his suggested equation is:

$$\frac{d}{d_{4.5}} = b_o \frac{h_d}{h} + b_1 \frac{h_d^2}{h^2} + b_2 \frac{h_d^3}{h^3} \tag{2.5}$$

According to these authors, both models were considered adequate for the data independently collected. It is worthwhile to say at this point that to some extent, the evaluation of a taper model involves a subjective judgement of the prediction ability of the function. What is considered a reasonable bias or acceptable residual by one investigator will not be necessarily true for another researcher.

In studying generalized taper functions, Newnham (1958) believed that a quadratic parabola was adequate for the description of the bole shape.

Prodan (1965) found that a satisfactory taper function should be:

$$\frac{d}{d_{1.30}} = \frac{(h_d/h)^2}{b_o + b_1 \frac{h_d}{h} + b_2 \frac{h_d^2}{h^2}} \tag{2.6}$$

In this equation, d is the predicted diameter at height h_d of a tree with total height h. The diameter at breast height ($d_{1.30}$) is measured in centimeters at 1.30 m from the ground level.

Munro (1966) developed a linear transformation of the quadratic paraboloid for the prediction of diameter inside bark (d_{ib}) at any given height (h_d) in feet from the ground.

$$d_{ib} = d_{4.5} \sqrt{b_1 + b_2 \frac{h_d}{h-4.5}} \qquad (2.7)$$

As in the previous models $d_{4.5}$ is the dbh at 4.5 feet from the ground and h is total tree height in feet. Kozak and Smith (1966) reported that Munro's model failed to describe the stem at the butt and tip but, after integration, satisfied many of the needs with respect to merchantable volume.

An improved model was suggested by Munro in 1968:

$$\frac{d^2}{d_{4.5}^2} = b_1 + b_2 \frac{h_d}{h} + b_3 \frac{h_d^2}{h^2} \qquad (2.8)$$

Here the symbols have the same meaning as in Munro's previous model. Diameters were overestimated in the upper third of tree height and Kozak et al. (1959) suggested an improvement for Munro's last model by imposing the condition that diameter be zero at the tip. The new model simplified to:

$$\frac{d^2}{d_{4.5}^2} = b_1 \left(\frac{h_d}{h} - 1\right) + b_2 \left(\frac{h_d^2}{h^2} - 1\right) \qquad (2.9)$$

In a study of taper and volume for red alder, Bruce et al. (1968) derived prediction equations expressing ratio of squared upper stem inside bark diameters to squared $d_{4.5}$ outside bark as a function of $d_{4.5}$, h, and the 3/2, 3rd, 32nd and 40th powers of relative length. This was necessary to obtain a model that could tentatively fit the several inflection points along the stem.

Demaerschalk (1971a) reported a logarithmic model:

$$\log d = b_o + b_1 \log d_{4.5} + b_2 \log h_d + b_3 \log h \qquad (2.10)$$

He suggested that by using this function, no conditioning was necessary to insure that the estimated diameter at the top be zero and that no negative diameter estimates occurred.

Demaerschalk (1971b) developed a compatible system based on the combined variable volume equation of Spurr (1952) and existing volume tables for British Columbia species. In conflict with those results, Evert (1971) found seriously biased estimates of true volume when both logarithmic and the combined variable approach were used.

A third-degree polynomial was suggested by Bennett and Swindel (1972) after an investigation of taper of planted slash pine:

$$d_h = b_1 \frac{d_{4.5}\,(h-h_d)}{h-4.5} + b_2 \cdot (h-h_d) \cdot (h_d-4.5) + b_3\, h(h-h_d)(h_d-4.5) + b_4\,(h-h_d)(h_d-4.5)(h+h_d+4.5) \tag{2.11}$$

That model is essentially a third-degree polynomial in respect to h_d and the results were considered adequate for diameter prediction with the data used. Characteristics of volume estimation with that model were not reported.[2]

Max and Burkhart (1976) applied the techniques of segmented polynomial regression to describe the stem of loblolly pine with a three models jointed function. The resulting model was used with an independent data set and when compared to a single quadratic taper function, the segmented polynomial was found superior.

A computerized system proposed by Demaerschalk and Kozak (1977) involved the use of two mathematical functions, one describing the upper

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[2] In this research project a third-degree polynomial was also used in a trial to express stem form. The independent variables of the model however, were defined differently. After unsatisfactory results were obtained, this author used principal component analysis.

bole and jointed at the inflection point with a second function for the lower portion. Biased diameter estimates at the butt level were detected by the authors, but the prediction of diameter inside bark was very accurate over most of the stem. That segmented model was also used by Waite (1977) with data obtained in northern Brazil with Gmelina plantations. Although good results were achieved for the estimation of diameter and volumes, that author concluded that the system of segmented equations originally proposed was not adequate for that species.

Cao (1978) reported a comprehensive review of taper functions where the efficiency of several previous and new models was compared. With respect to Demaerschalk and Kozak's study, Cao pointed out that the complexity of the system is the main drawback that makes it impractical.

To summarize, all investigators tried to describe tree taper with different models of varying degree of complexity. It is clear that each and every model produced biased or undesirable results when used out of the range of data upon which its development was based. The research studies are conflicting in their conclusions and there is yet no agreement on what should be the model for a reasonable universal taper function, if any.

The Use of Principal Component Analysis

Polynomial regression analysis is just a special case of multiple regression analysis. Since the idea of multidimensional sources of variation has been present in the previous attempts to express stem form of trees, the investigation of the problem with a more flexible multivariate technique is reasonable.

Multivariate statistical analysis is concerned with data collected on several dimensions of the same individual, i.e. several variates (Morrison, 1976).

Principal component analysis (PCA) is one of the techniques under the general heading of multivariate methods. PCA is extremely useful for reduction of data dimensionality, analysis of the structure of multivariate systems and for descriptive purposes. It was first developed with the variance maximizing studies of Hotteling (cited from Morrison, 1976).

Fries (1965) and Fries and Matern (1966) introduced the use of principal component analysis in the investigation and development of tree taper curves. After an analysis of those studies, Kozak and Smith (1966) claimed that the use of simple functions, sorting and graphical methods, is adequate for many uses in forest operations and research. Of course the definition of what is an adequate prediction characteristic of a taper model must be considered in the decision process of what technique shall be used.

Evert (1971) mentioned some of the limitations involved in the attempts to increase the precision of volume estimate of standing trees. With regard to principal component analysis he had the following comment:

> A lot of progress has been made, using multivariate functions in terms of dbh and height only, or using measurements of upper-stem diameters as well . . . Unfortunately, obtaining solutions to multivariate functions by regression analysis is not without pitfalls. One of these pitfalls is possible bias resulting from the preparation of tree-volume equations from data already expressed as equations or as freehand curves (p. 353).

The data already expressed as equations that Evert refers to, are the eigenvector elements associated with the eigenvalues. However, it must be observed that the eigenvalues contain all the variability of the original data matrix, and can not be a source of bias.[3] In fact bias will be introduced if an inadequate model is used to relate the eigenvector elements to the relative position of the original diameter measurements along the stem of the sample trees. That is, biases are due to the model chosen, not to the technique or, in this case, to the data.

Principal component analysis was used by Liu (1973) in the development of a taper model for loblolly pine data obtained from Crown Zellerbach Company. The developed function was reported to be an effective diameter predictor for any height along the stem regardless of tree size.

Liu and Keister (1978) described the procedure by which PCA was applied to define stem taper of loblolly and slash pine. A single eigenvalue absorbed 99 percent of the total variance of the data (diameter measurements collected at the same proportional heights for all trees). To obtain a taper model, regression analysis was used, the first eigenvector being the dependent variable, and the corresponding proportional heights and their powers the independent variables. Total and merchantable volume functions were derived by integration of the taper model. No significant difference was observed when actual volume of sample trees was estimated by using both the volume function obtained and Smalian's procedure.

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[3] The terminology is more completely explained in the description of the statistical method, Chapter III, p.

Taper and Volume Systems

A comparative analysis of different compatible systems of taper and volume was provided by the study of Munro and Demaerschalk (1974). Taper based and volume-based systems were defined, and from that study, the following statement is pertinent: "A good tree taper and volume system should be unbiased for all tree sizes in the estimation of diameter at any height, height for any diameter, volume of any section and total tree volume" (p. 198).

A taper and volume system is called taper-based when volume is obtained after an adequate taper function is defined. On the other hand, the system is called volume-based if a taper model is derived from an existing and accepted volume function. To be considered a compatible system, volume obtained after the integration of a taper function must be equal to the actual volume also estimated, but by using a traditional procedure. Currently, it is a broadly accepted philosophy that the development of a compatible taper-based system of taper and volume must achieve the desirable characteristics synthesized with Munro and Demaerschalk's statement.

However, the investigation of tree stem form is complex, and Demaerschalk (1973b) has shown that an equation considered best for taper is not necessarily best for volume. This statement needs an appropriate interpretation. In agreement with Demaerschalk, this author adds: a bias considered not significant for diameter prediction may have different effects when volume is computed by integration of a given taper model. That is, volume is affected differently if that bias occurs in the upper sections of the stem or close to the ground. However, unbiased volume estimates for any section of the stem will only be obtained if an accurate taper model is defined. Hence, the degree of accuracy of the taper model must be properly identified previous to the volume computation.

After an elaborate review of stem form, Grosenbaugh (1966, p. 456) recognized the complexity of the subject and concluded: "Quantification of tree form requires measurement of tree heights and diameters along as much of the stem as possible. No single pair of coordinates is of much value in assessment of tree form."

In a stem form investigation, however, several diameters are measured along the stem of sample trees. In the model building process, those measurements are used in a way such that the taper function is developed upon those diameters and heights but, for practical uses, the expression is a function of dbh and tree total height.

CHAPTER III

MATERIAL AND METHODS

Study Area and Data

The tree measurement information for this study consists basically of 188 stems obtained from four compartments planted with loblolly pine at Irati National Forest in southern Brazil (Figure 2). The description of this site is summarized as follows (Golfari, 1978):

Irati National Forest, Irati County, State of Parana, Brazil

Latitude: $25^{o}27'S$

Longitude: $50^{o}35'W$

Altitude: 850 m (2,790 feet)

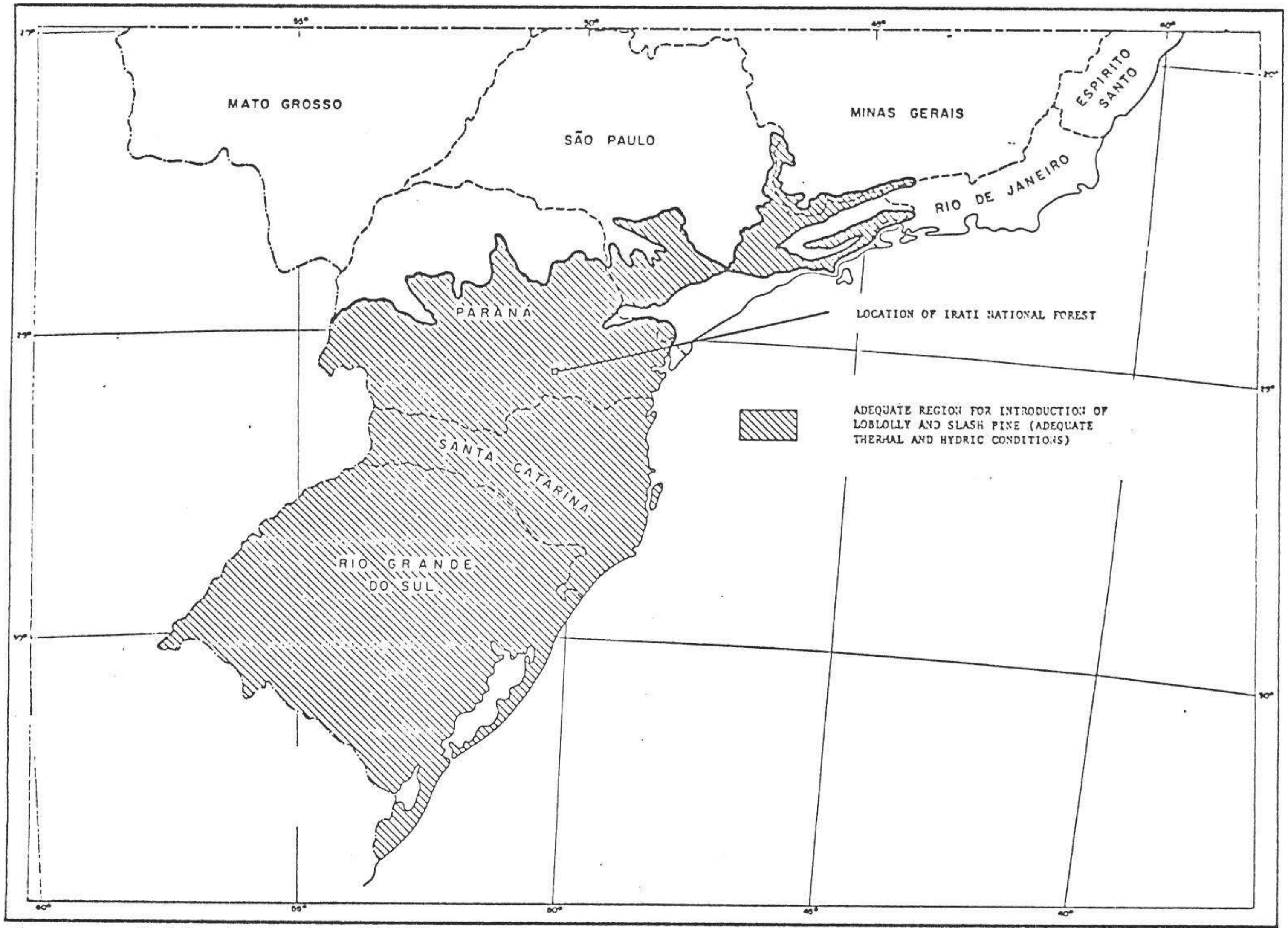
Climate: annual precipitation . . . . . . 1442 mm (58 inches)

mean annual temperature . . . . $17.2^{o}C$ ($63^{o}F$)

Soil: dark reddish latosol

Table I is the joint frequency distribution of the sample trees with respect to dbh and height. Volume of trees has a nearly linear association with height but is greatly related to the square of diameter at breast height. Thus sample trees were randomly collected within each diameter class rather than randomly distributed over the entire target population. Age range of the data was from 5 to 19 years.

The so-called Hohenadl's method of tree measurement and volume estimation (Hohenadl, 1936) was used in providing the data needed for this study. Appendix B contains a review on the method as well as a



Source: Golfari et al., (1978, p. 45)

Figure 2. Location of Irati National Forest, State of Parana, Brazil

TABLE I

TREE DATA FREQUENCY DISTRIBUTION

Diameter Class (cm)	Height Class (m)												Total
	4	6	8	10	12	14	16	18	20	22	24	26	
6	7	1											8
8	5	4		2									11
10		9	1										10
12		2	3	5	1								11
14			4	4	2	1							11
16					4	1	3	2					10
18					2		4	5					11
20					2	1	1	5	1				10
22					1		2	6	1				10
24							1	5	3	1			10
26								2	7	1			10
28								1	9				10
30								3	5	2		1	11
32									3		8		11
34									1	4	6		11
36										2	7	2	11
38										2	5	4	11
40											7	4	11
TOTAL	12	16	8	11	12	3	11	29	30	12	33	11	188

Diameter class 6 cm = 5.0 -- 6.9 cm
Height class 4 m = 3.0 -- 4.9 m

detailed description of the field procedure used. In essence, the data obtained from each sample tree were: $d_{1.30}$ (diameter at breast height, outside bark), total height, and beginning from the tip, diameter inside bark at 1/10, 3/10, 5/10, 7/10, 8.333/10, 9/10, and 9.667/10 along the stem above a 15.0 cm standard stump height. Hence, all taper variability was condensed in a 7 x 188 matrix: seven positional diameters and 188 trees. The field measurements originally recorded on forms were punched on cards and SAS-79 edition was used all the way through the data processing and statistical analysis.[1]

An Attempt With Polynomial Analysis

It was believed that a third degree polynomial of the form $Y = b_o + b_1 x + b_2 x^2 + b_3 x^3$ could fit the data because the plot of some cubic polynomials appears to follow a taper much like a tree stem.

It must be recalled that all polynomials used in previous investigations of stem form were developed in attempts to predict diameter at any point along the stem as a function of dbh, tree total height and the absolute position of the diameter being predicted. In this study however, the third degree polynomial was suggested to predict radii (diameter) as a function of its relative position along a stem of unit length. This approach has produced biased estimates for both diameter and volume.

The basic idea was to reduce geometrically the dimensions of all trees in order to relate all radii (diameters) and their absolute position to the smallest tree (Figure 3). That is to say, all measurements

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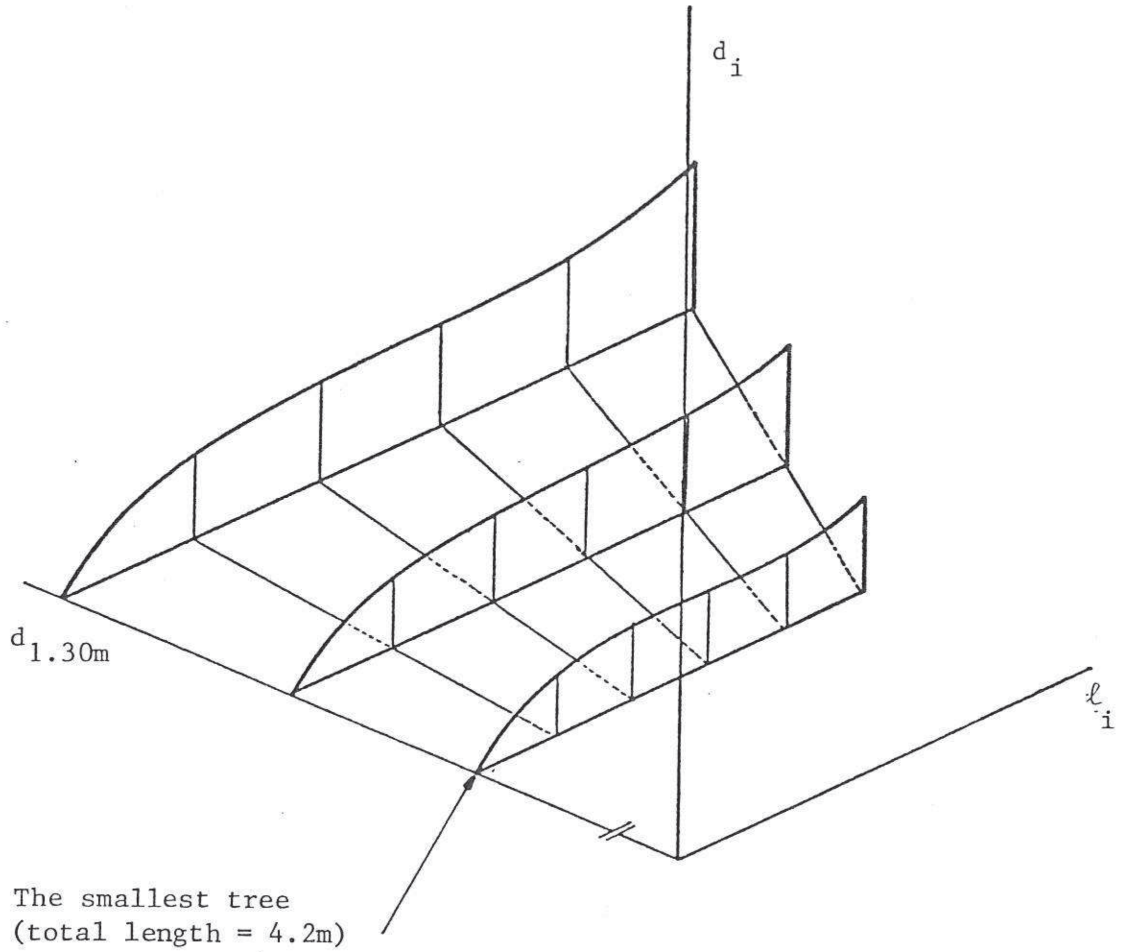
[1] Statistical Analysis System, SAS-79 edition is a collection of subroutines for data analysis prepared by SAS Institute Inc., Raleigh, NC.



ℓ = 0.10L, 0.30L, ... , 0.967L

Figure 3. Schematic Representation of Normalized Diameter Data Used in the Third-Degree Polynomial Regression Analysis

were normalized. After a taper model was satisfactorily fitted to these standardized values, the function was expanded back to the original data by using a conversion factor. The assumption on which this procedure was based is that, regardless of size, all trees have exactly the same form, and that the inflection point is located at a constant relative position.

That proved to be a wrong assumption. Volumes were overestimated for small trees and underestimated for larger trees. In trying to find alternatives to fit the data, an important conclusion was drawn: a second degree polynomial, or parabola, could do as well. The addition of the third power element to the model does provide a better fit for the butt region of the stems of many species. However, loblolly pine stems within this age range of 5 to 19 years old, do not have a pronounced butt swell and trees are more cylindrical at the base. In addition all measurements were done above a 15.0 cm stump height and this source of taper, if present, was partially eliminated. This conclusion was later useful in the selection of a model for the principal component analysis of the data.

Principal Component Analysis

The principal aim of multivariate statistical methods is to have a better understanding of the underlying structure of the data represented with a joint frequency distribution of several variates (Marriot, 1974).

Among several statistical techniques under the general expression of multivariate methods, principal component analysis is useful in the ordination of variables as an aid for the interpretation of multivariate data.

It must be emphasized that principal component analysis is not directly used for hypothesis testing. It is just an exploratory technique for an efficient evaluation of the dimensions of variability. This is achieved with the development and analysis of orthogonal components.

Suppose a data vector of p random variables is measured for n independent observations so that an p x n data matrix X is obtained:

$$X = \text{Row } i \overset{\text{Col. } j}{\begin{bmatrix} x_{11} & \cdots & x_{1p} \\ \vdots & \vdots & \\ \cdots & x_{ij} & \\ \vdots & & \\ x_{n1} & \cdots & x_{np} \end{bmatrix}}$$

$$i = 1 \ldots n$$
$$j = 1 \ldots p$$

The method of principal component analysis consists of transforming a set of correlated variables $X_1 \ldots X_p$ to a new set of uncorrelated variables or orthogonal components $C_1 \ldots C_p$, observing the following basic properties (Marriot, 1974 and, Isebrands and Crow, 1975):

a. each new variable c_k is a linear combination of the original variables X_j, $j = 1, \ldots p$.

$$c_k = a_{k1} X_1 + a_{k2} X_2 + \ldots + a_{kp} X_p$$

b. the sum of squares of the coefficients equals unity. That is,

$$\sum_{j=1}^{p} a_{kj}^2 = 1, \text{ for each } k = 1, \ldots, p.$$

c. the new variables are uncorrelated with each other.

d. the total variation among the new variables is equal to the total variation among the original variables.

e. the variance of each new variable decreases in order, i.e. of all possible linear combinations, C_1 has the largest variance.

Of all possible linear combinations uncorrelated with c_1, the one with the largest variance is c_2, and so forth.

With this linear and orthogonal transformation, a new set of p variables is obtained, uncorrelated with each other and arranged in order of decreasing variance (Marriot, 1974). The value of this new matrix however, is that a relatively small number of new variables or principal components may account for most of the total variance of the original data. Hence, the vector of coefficients or eigenvector associated with these new variables may be considered separately to express the multidimensional structure of the original data.

The solution for this algebraic procedure is equivalent, in matrix algebra, to the determination of the eigenvalues (characteristic roots or latent roots) and the associated eigenvectors (characteristic vectors) of the p x p variance-covariance matrix developed from the original data. The eigenvalues turn out to be the variances of the respective principal components, while the eigenvector elements provide the coefficients for obtaining the principal components.

Taper Development

After the polynomial approach produced unsatisfactory results, principal component analysis was used to investigate the stem form structures of the data and to define a mathematical expression of that structure. This multivariate statistical technique was applied following basically the same outline of those studies referred to in the review of previous work done on the subject.

Because the taper function was ultimately converted by integration to a volume function, all the computations involved in the taper model development were done using radii inside bark instead of diameters inside bark.

Having prepared the data in a two-dimensional array of seven positional radii (diameters) measured along each of 188 stems, a symmetric 7 x 7 sum of squares and cross-products matrix was obtained using PROC CORR (Statistical Analysis System, SAS/1979 edition).

$$D = \text{(Data Matrix)} \quad \begin{array}{c} \text{Stem no.} \\ 1 \\ 2 \\ \cdot \\ \cdot \\ \cdot \\ 188 \end{array} \begin{array}{c} \text{Radius no.} \\ \begin{array}{cccc} R_1 & R_2 & \cdots & R_7 \end{array} \\ \begin{bmatrix} X_{1,1} & X_{1,2} & \cdots & X_{1,7} \\ X_{2,1} & X_{2,2} & \cdots & X_{2,7} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ X_{188,1} & X_{188,2} & \cdots & X_{188,7} \end{bmatrix} \end{array}$$

$$P = \text{(sum of squares and cross-products matrix)} \quad \begin{array}{c} \\ R1 \\ R2 \\ \vdots \\ R_7 \end{array} \begin{array}{c} \begin{array}{cccc} R_1 & R_2 & \cdots & R_7 \end{array} \\ \begin{bmatrix} SS_{R1} & CP_{R1,R2} & & CP_{R1,R7} \\ & SS_{R2} & & \\ & & \ddots & \vdots \\ & & & SS_{R7} \end{bmatrix} \end{array}$$

where SS_{R1} is the sum of squares for R1, $CP_{R1,R2}$ is the sum of cross-products of R_1 and R_2, and so forth.

The solution for the eigenequation $|P - \lambda I| = 0$ was computed by using PROC MATRIX and the SAS commands EIGEN, VALS and VECS. This produced a diagonal matrix of eigenvalues ranked in decreasing order and, an associated matrix of eigenvectors, each column vector being related to an eigenvalue.

It was observed that the first eigenvalue by itself accounted for 99.72 percent of the total variance. This was considered to be a very strong evidence that the first eigenvalue and its associated eigenvector were extremely useful in explaining the latent structure of the data. Following suggestions of previous studies (Fries and Matern, 1966; Liu, 1973) and for interpretation purposes, the eigenvector elements associated with the three first ranked eigenvalues were plotted against the corresponding relative position u along the stem (Figure 4). After observing the trend of these three eigenvectors it was concluded that only the first eigenvalue was sufficient to explain and describe the average stem form of the stems considered in the analysis.

Polynomial regression analysis was then used to fit a model to the data represented by the seven elements (ELT) of the first eigenvector as a function of their relative position (u) along the stem of length unit. PROC RSQUARE was used to obtain all possible models with the following independent variables: $u^{1/4}$, $u^{1/3}$, $u^{1/2}$, u, $u^{3/2}$, u^2, $u^{5/2}$ and u^3. Among all possible combinations of those variables, five models were selected because of their very high coefficient of determination, and PROC GLM was used with each model; once with the intercept b_o included in the least squares solution, and a second run forcing the model to pass through the origin. All ten models thus obtained were similar in their eigenvector elements prediction ability, but the following model was selected:

$$\hat{ELT} = F(u) = b_o + b_1 u^{1/4} + b_2 u^{1/3} + b_3 u^{1/2} + b_4 u \qquad (3.1)$$

Hence, among all models evaluated, equation 3.1 was the best function to fit the average taper represented with the eigenvector elements

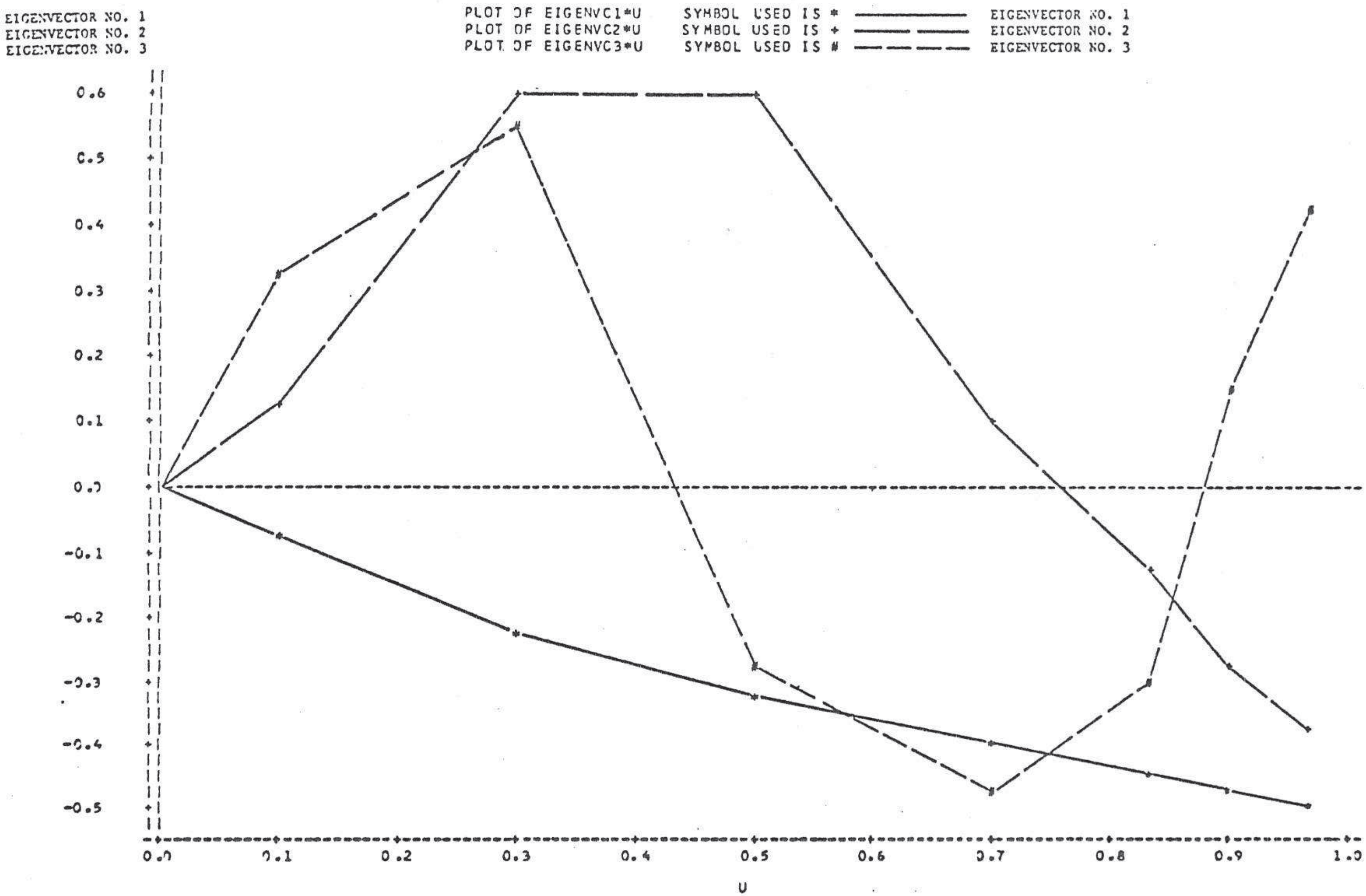


Figure 4. Plot of Eigenvector Elements for the Three First Eigenvalues Against Relative Position Along the Stem

associated with the first eigenvalue. Because of the computational nature of the procedure however, that model will only estimate relative radii of a stem of length unit, i.e. related to the relative position of the eigenvector elements. In order to have that model expressing predicted radii (diameters) as a function of actual length, a transformation of the independent variables was introduced and the model became:

$$\hat{r}(\ell) = F(\ell/L) = b_o + b_1(\frac{\ell}{L})^{1/4} + b_2(\frac{\ell}{L})^{1/3} + b_3(\frac{\ell}{L})^{1/2} + b_4(\frac{\ell}{L}) \quad (3.2)$$

This is the taper model for the prediction of a radius $\hat{r}(\ell)$ (diameter) at the absolute position ℓ from the tip of a stem with average form and total length L above a 15.0 cm stump height.

The objective of this study implies however the development of a taper model for the prediction of inside bark of radii (diameters) along the stem of any tree considered in the analysis and not only those with stem of average form. Therefore, suppose a tree has a radius $r_{1.30}$ measured at 1.30 m from the ground, and has a stem of length L measured above a 15 cm stump height. The previous model needs an adjustment factor that will force the function to pass through the point of the dbh. This factor is the ratio between the inside bark actual radius at the dbh, $r_{1.30}$, and the predicted radius at the same point $\hat{r}(\ell_{1.30})$.

The final taper model obtained was:

$$\hat{r}(\ell) = F(\ell) = \frac{r_{1.30}}{\hat{r}(\ell_{1.30})} \cdot \hat{F}(\ell/L) \quad (3.3)$$

where $\hat{r}(\ell)$ = predicted radius inside bark of an absolute position from the tip, in meters

$r_{1.30} = d_{1.30}/2$ (inside bark, in meters)

$\ell_{1.30} = L - 1.15$ m

$$\hat{r}(\ell_{1.30}) = b_0 + b_1\ell_{1.30}^{1/4} + b_2\ell_{1.30}^{1/3} + b_3\ell_{1.30}^{1/2} + b_4\ell_{1.30}$$

$\hat{F}(\ell/L)$ = equation 3.2

Volume Estimation

Total volume inside bark was predicted by integration of equation 3.3 as follows:

$$\hat{V} = \int_0^L \pi \, (\hat{r}(\ell))^2 \, d\ell$$

$$\hat{V} = \int_0^L \pi \cdot \left[\frac{r_{1.30}}{\hat{r}(\ell_{1.30})} \cdot \hat{F}(\ell/L) \right]^2 d\ell$$

$$\hat{V} = \pi \cdot \left[\frac{r_{1.30}}{\hat{r}(\ell_{1.30})} \right]^2 \cdot \int_0^L \left[\hat{F}(\ell/L) \right]^2 d\ell \qquad (3.4)$$

In order to have a measure of the prediction ability of the volume function (equation 3.4) obtained by integration of the taper function (equation 3.3), total volume inside bark was also estimated by using a "traditional procedure". That was the so-called Hohenadl's method of stem volume estimation.

The data used for the taper development was originally collected according to Hohenadl's method, and Appendix B contains a detailed description of the field and office procedures involved in that method. Formulas for the volume estimation are also given.

Total volume inside bark was estimated for all 188 trees by using both procedures: the taper derived volume function and Hohenadl's method. Analysis of correlation, PROC CORR, was applied to observe the degree of association between these two volume estimates. Residuals of the volumes predicted with the taper-derived equation over those estimated with Hohenadl's method were also obtained and analyzed.

CHAPTER IV

RESULTS AND DISCUSSION

Eigenvalue and Eigenvector Analysis

The solution for the principal component analysis - PCA, requires an interpretation of the eigenvalues and associated eigenvectors.

Because the data matrix consisted essentially of diameter measurements along the stems of loblolly pine sample trees, the latent or hidden structure of that matrix is that of stem form. PCA assumes that the observable or manifest variates can be expressed in a smaller number of latent factor variates, the principal components (Morrison, 1976). In this study there were seven manifest variates, the original seven radius (diameter) measurements, thus seven orthogonal components were obtained, of which ideally just one or very few are of interest, the principal components. The importance and utility of a principal component can be measured by the proportion of the total variance attributable to it.

The eigenvalues or variances of the seven orthogonal components related to the average stem form are listed in Table II. The associated eigenvectors are recorded in Table III. Because of the orthogonality property, each eigenvalue can be considered separately. Observing the cumulative percentage of variance explained, it is obvious that after the contribution of the first eigenvalue, 99.72 percent, the additional variance accounted for by all other eigenvalues is negligible. That is

TABLE II

EIGENVALUES OF THE SUM OF SQUARES AND CROSS-PRODUCTS MATRIX

EIGENVECTOR NUMBER	EIGENVALUE	EIGENVALUE PERCENT	CUMULATIVE EIGENVALUE PERCENT
1	10.0236	99.72	99.72
2	0.0167	0.17	99.89
3	0.0041	0.04	99.93
4	0.0024	0.02	99.95
5	0.0019	0.02	99.97
6	0.0015	0.02	99.99
7	0.0012	0.01	100.00
TOTAL	10.0514	100.00	

TABLE III

EIGENVECTORS ASSOCIATED WITH THE EIGENVALUES OF THE SUM OF SQUARES AND CROSS-PRODUCTS MATRIX

ELEMENT NUMBER	EIGENVECTOR NUMBER						
	1	2	3	4	5	6	7
1	-0.073747	0.127024	0.313448	0.296600	0.405945	-0.790901	0.043459
2	-0.213371	0.611514	0.555317	-0.426355	0.155088	0.257049	-0.015540
3	-0.330984	0.599204	-0.282757	0.531880	-0.376848	0.029875	0.160156
4	-0.409884	0.102944	-0.465203	-0.378004	0.054499	-0.277370	-0.618217
5	-0.447714	-0.131427	-0.312069	-0.276361	0.329024	0.001063	0.707290
6	-0.464723	-0.273530	0.150432	0.468774	0.430824	0.439653	-0.296533
7	-0.505038	-0.384944	0.420703	-0.091333	-0.610169	-0.193077	0.042851

to say that the first eigenvalue is sufficient to explain stem form of the data. That evaluation however, involves some subjectivity, and a stronger basis for that conclusion was obtained with the analysis of the plot of the eigenvector elements associated with the three first ranked eigenvalues. It can be observed in Figure 4, that the trend of the eigenvector elements associated with the first eigenvalue was closely related to stem taper, while those of the second and third eigenvalues had no apparent meaning for this purpose.

After using the first eigenvector to explain stem form of white birch, Scots pine and lodgepole pine, Fries (1965) found the elements of the second eigenvector correlated with crown/stem length ratio, and the third eigenvector elements explaining dbh. Liu (1973) tried to improve the results of his study by applying PCA to groups of trees classified by size. After the solution of the eigenequation, he found no need to separate trees into diameter classes because of the small improvement on the cumulative percentage of the eigenvalues.

The alternative of grouping the data by tree size was not used in this study given that the taper variability explained with the first eigenvalue was considered to be extremely high. Thus a taper model developed only upon the first eigenvalue would produce the average stem form, applicable to all of the trees within the data range considered.

Model Building

Because of the high proportion of taper variability explained with the first eigenvalue, the elements of its associated eigenvector can be analyzed as being in the same proportion as the radii (diameters) of a tree with average form. Multiple regression analysis was used to fit a polynomial function to the elements (ELT) of that first eigenvector (Figure 4), and among the models tested, the following was found to be a good relative diameter predictor:

$$\hat{ELT} = b_o + b_1 u^{1/4} + b_2 u^{1/3} + b_3 u^{1/2} + b_4 u \quad (3.1)$$

where $\hat{ELT}$ = predicted eigenvector element or relative radius for a tree of unit length and average form.

u = relative position along a stem of unit length. (0.10, 0.30 ... 0.967)

b_0 = 4.3106887 E-06

b_1 = 3.10149948

b_2 = -6.65976505

b_3 = 4.68992834

b_4 = -0.62829584

The observed coefficient of determination (R^2) was 0.9987. Relative radius prediction ability was simulated and that model produced a good fit all along the stem of unit length, including the butt region, where most of the previous studies have shown biased estimates.

However, the objective of this study required a model for the prediction of actual diameter inside bark (radius) for trees of any actual length and with taper that deviates from the average stem form (within the data range). Hence, two transformations were included in equation 3.1. First, the independent variables were expressed as a ratio of the absolute position of the diameter being predicted, and stem total length (Equation 3.2). Next, the form adjustment was done by multiplying the radius predicted for a tree of average form, by the ratio between actual $r_{1.30}$ and the predicted $r_{1.30}$ for that tree (Equation 3.3).

The resulting equation thus obtained (Equation 3.4), is basically a taper function for diameter inside bark (d_ℓ) prediction at any position ℓ from the tip of stems with total length L = h - 0.15 m. Obviously this statement is true only for the species, dbh and h range, and site considered for this study.

Volume Estimation

Volume estimates were obtained for all 188 sample trees with the volume function (Equation 3.4) developed by integration of the taper function. Volumes were also predicted using Hohenadl's method. Correlation analysis was carried out in order to observe how consistent both procedures were in their volume prediction ability. The observed Pearson's coefficient of linear correlation was equal to 0.9927, which reflects an extremely high degree of association for the two variables being considered. However, that only says that the distribution of both estimates is consistent over the data range. It could happen that volume estimates obtained with both procedures "consistently" diverged over the data range, and the coefficient of linear correlation would still be the same.

In order to help in the interpretation of such associative pattern, both volume estimates were simultaneously plotted against $d_{1.30}$, a measure of stem size (Figure 5). The distribution of predicted volumes showed no evidences that two populations could be identified. However, although helpful, that is just a graphical representation of the volume estimates obtained, and a numerical analysis of the residuals was considered to be adequate in the evaluation of the prediction ability of the taper-derived volume function.

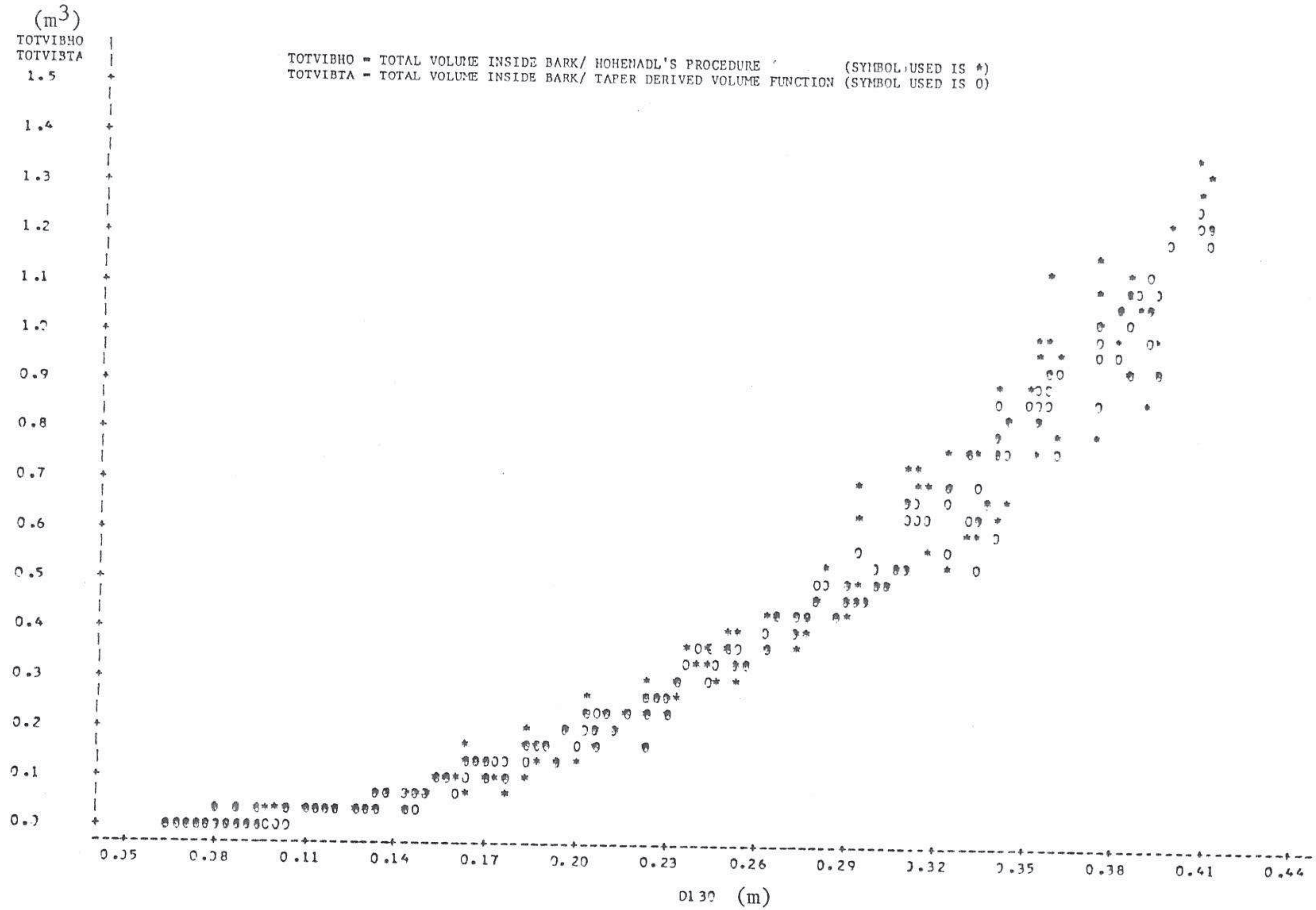


Figure 5. Plot of Total Volume Inside Bark Estimates Obtained With the Taper-Derived Volume Function and With Hohenadl's Procedure Against dbh

Residuals were analyzed by expressing the differences between the two volume estimates as a percent of the volume predicted with Hohenadl's method. The distribution of the percent residuals thus obtained over the diameter at breast height range is displayed with Figure 6. A similar trend was also observed with the distribution of residuals over the height range. It is noticeable (Figure 6) that for small trees, those with $d_{1.30}$ between 6.0 and 12.0 cm, volumes were overestimated with the taper-derived volume function as compared to Hohenadl's method. No such divergence was observed for the predicted volume of trees with $d_{1.30}$ larger than 12.0 cm. In an overall analysis the percent residuals ranged from -30.60 to 28.50, which was considered adequate for the objectives of this study. If more diameter measurements were obtained during sampling, then more accurate and reliable estimates would be determined with both methods, thus reducing the percent residual range.

It is worthwhile to note the fact that volume estimates obtained with Hohenadl's method will always be more accurate with decreasing stem height. The smaller the sections into which a stem is divided the more accurate will that method be in the total volume estimation.

On the other hand, the taper function obtained through principal component analysis was the expression of average stem form. The inclusion of an adjustment factor was done in order to have that taper function fitting all stems sampled. The solution for the eigenequation led to the conclusion that the statistical procedure was efficient to express stem form structure of the data.

These conflicting measures of efficiency observed for both procedures allows us to interpret the diverging volume estimation of small trees as being a result of stem form that seriously deviated from average

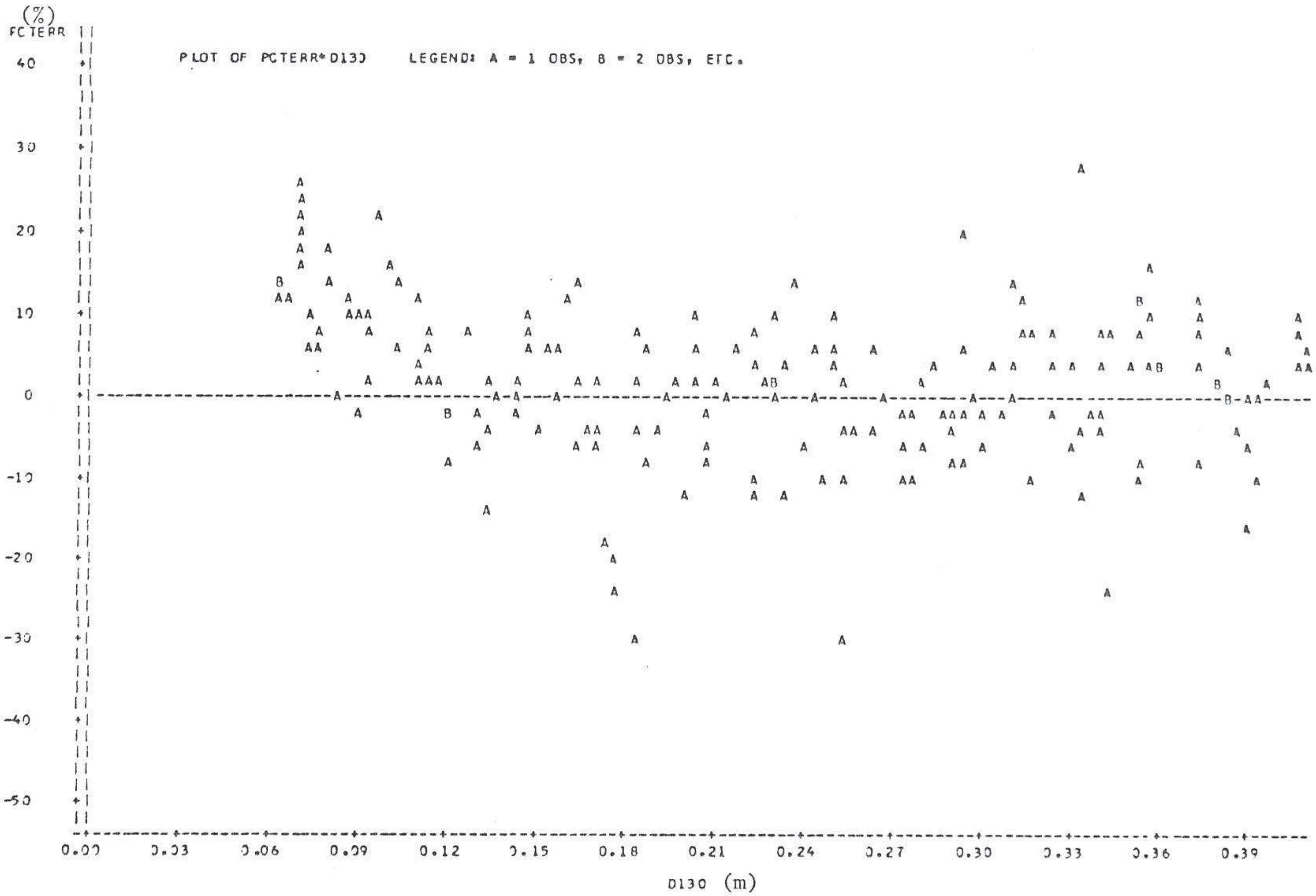


Figure 6. Distribution of Residuals of Taper-Derived Total Volume Inside Bark Estimates

stem form. However, the value of the system must be evaluated mostly in regard to those trees considered in a size range of economical importance. In this respect, more relevant for practical purposes, the taper-derived volume function was found to be an adequate predictor that agreed well with volume estimates obtained using Hohenadl's method.

A complete listing of the volume estimates determined with both Hohenadl's method and the taper derived function is given in Appendix C. Field measurements of all sampled stems are also displayed.

CHAPTER V

SUMMARY AND CONCLUSIONS

The purpose of this study was essentially the investigation of stem form of planted loblolly pine and its analytical expression with a mathematical function. By definition, integration of such a developed taper model provides a volume function.

Data were obtained in a loblolly pine plantation located in southern Brazil. A modified Hohenadl's method of tree stem measurement was used to collect the data needed for the analysis. Diameters inside bark were measured at seven relative positions along each of 188 stems.

Previous stem studies were mostly based on simple models or on the use of polynomials. Several of those investigators tried to define a universal taper model, but conflicting results were reported. On the other hand, those few studies on which principal component analysis was used, have demonstrated consistency in their conclusions. This refers to the algorithm, not to the structure of the final equation itself.

In this research study, a first attempt was made using a third-degree polynomial. However, results were biased and discouraging. Alternatives to improve that taper equation were initially considered, such as the inclusion of an additional independent variable to express the inflection point, or grouping the data by size. Nevertheless, that procedure was abandoned and a multidimensional and more flexible approach was finally used.

The objective of this study was successfully achieved with the use of principal component analysis, a multivariate exploratory technique.

The procedure consisted of solving the eigenequation of the sum of squares and cross-products matrix obtained from the data matrix of seven positional radii (diameters) and 188 stems. The solution for the eigen-equation provided seven eigenvalues or latent roots, and seven associated column eigenvectors. Performing that matrix solution is algebraically equivalent to transforming a set of p correlated variables into a new set of p uncorrelated variables.

In interpreting the seven eigenvalues obtained, only the first was considered significant to explain stem form, the latent structure of the data. Thus the eigenvector of seven coefficients or elements associated with that first eigenvalue was considered to be an expression of average stem form for stems of unit length. Polynomial regression analysis was used to fit a model to those elements of the first eigenvector. The taper function thus obtained was adjusted to fit the taper of trees with any length and any dbh. The final developed model was;

$$\hat{r}(\ell) = \frac{r_{1.30}}{\hat{r}(\ell_{1.30})} \cdot \hat{F}(\ell/L) \tag{3.3}$$

where

$\hat{r}(\ell)$ = predicted radius inside bark at any absolute position ℓ from the tip, in meters.

$r_{1.30}$ = $d_{1.30}/2$ (inside bark, in meters)

$\ell_{1.30}$ = L - 1.15 m

$$\hat{r}(\ell_{1.30}) = b_o + b_1\ell_{1.30}^{1/4} + b_2\ell_{1.30}^{1/3} + b_3\ell_{1.30}^{1/2} + b_4\ell_{1.30}$$

$$\hat{F}(\ell/L) = b_o + b_1(\frac{\ell}{L})^{1/4} + b_2(\frac{\ell}{L})^{1/3} + b_3(\frac{\ell}{L})^{1/2} + b_4(\frac{\ell}{L})$$

L = stem total length (meters) above a 15.0 cm stump height.

Total volume inside bark was predicted for all 188 sample stems by using both procedures: the volume function obtained by integration of the taper model and with Hohenadl's method.

Percent residual of the volume estimates obtained with the taper-derived function over those determined with Hohenadl's procedure, ranged from -30.60 to 28.50. The magnitude of the residuals as well as their consistent distribution over most of the data size range, were considered adequate for the purpose of this study.

After having evaluated the results obtained it was concluded that principal component analysis has proven to be an efficient means for the investigation of stem form. The results indicated that the objective has been met satisfactorily. Accuracy of the prediction ability of the taper-volume system developed can be significantly improved by measurement of diameters at a larger number of relative positions, perhaps 10 or 20.

Actually only total volume inside bark was computed. However, the main justification for the development of a taper-volume system is its flexibility in regard to the volume prediction for any section of the stem. By that means, merchantable volume estimates between any two length limits may be accordingly determined.

Among the previous stem form studies observed in the literature, some investigators tried to develop a "universal taper function". However, such a model must take in account all possible sources of stem form variability: among others, species, age, site, stand density and past management practices such as pruning and thinning. It is evident that this is an extremely difficult task. Rather than seeking a "cooking formula", this author believes that much better and efficient results will be obtained by grouping the data as much as is possible and convenient. This must be done by considering both the cost of the taper-volume system development and the benefits of its use in practice. In this context, principal component analysis is a technique of great potential when the reduction of data dimensionality is desirable.

A universal taper function will not be produced by principal component analysis. In the taper model building process, the number and nature of the independent variables may vary with each new data set. However, if properly applied and interpreted, this multivariate exploratory technique may be considered as an adequate algorithm for the investigation of stem form under most of the circumstances.

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APPENDICES

APPENDIX A

COMMON AND SCIENTIFIC NAME OF THE CITED SPECIES

Black spruce	(Picea mariana Mill.)
Gmelina	(Gmelina arborea L.)
Loblolly pine	(Pinus taeda L.)
Lodgpole pine	(Pinus contorta Dougl.)
Monterey pine	(Pinus radiata D. Dom.)
Norway spruce	(Picea excelsa Link)
Parana pine	(Araucaria angustifolia (Bert) O. Ktze)
Red alder	(Alnus rubra Bong.)
Red spruce	(Picea rubens Sarg)
Scots pine	(Pinus silvestris L.)
Slash pine	(Pinus elliottii Engelm.)
Sugar maple	(Acer saccharum Marsch.)
White birch	(Betula papyrifera var. communata Fern.)

APPENDIX B

THE HOHENADL'S METHOD

Hohenadl (1936) suggested a procedure for stem volume estimation. This appendix will introduce the original method proposed, review some studies related to that procedure and describe the modified method used in this study.

The Original Method

The procedure as it was first proposed, divides the stem into five or ten sections of equal length. The volume of each section is estimated according to Huber's formula, each section being treated as a cylinder and the corresponding volume obtained by:

$$V = L_H \cdot g_m = L_H \cdot \frac{\pi}{4} \cdot d_m^2 \qquad (B.1)$$

where g_m and d_m are respectively the basal area and the diameter at the midpoint of a section of length L_H.

For the total stem volume above stump height, five sections being considered (Prodan, 1951), the procedure is defined with the model:

$$V = 0.2L \cdot \frac{\pi}{4} \cdot (d_{0.1L}^2 + d_{0.3L}^2 + d_{0.5L}^2 + d_{0.7L}^2 + d_{0.9L}^2) \qquad (B.2)$$

where 0.2L is the length of each section or 2/10 of the total stem length, and $d_{0.1L} \ldots d_{0.9L}$ are the diameters measured at the midpoint of each section. Length is measured from top downward. Both inside and outside bark volumes may, evidently, be computed by using inside or outside bark diameter measurements.

Comments on the Method

In a description of a study conducted by Zimmerle (1949) with stems profile of Norway spruce, Assman (1970) pointed out that by using Hohenadl's method, all diameters are taken at the same relative position of the stem. An extremely regular pattern of form for stems of different length was observed. An additional advantage was a very low variability of the natural form-factor among trees (based on $d_{0.9L}$) when compared to the artificial form-factor (determined with $d_{1.30m}$). The same opinion is reported by Loetsch et al. (1973).

> Sectioning into equal relative lengths has the advantage that stems of different absolute length can be compared, which is not the case with sections of equal absolute length. It is an advantage of the latter method that long (which means mostly also big) trees will be measured more accurately than short trees. For accurate form investigations however, the volume determination of sections of equal relative length is preferable (p. 147).

Working with Parana pine in southern Brazil, Hoogh, et al. (1978) observed the same trend and stated that the artificial form-factor $f_{1.30m}$, depends on the $d_{1.30m}$ and consequently on the height of the tree.

Heger (1965) described Hohenadl's original method as well as the study of Krenn and Prodan (1944) where the method is also analyzed. Heger pointed out that the method represents a synthesis of both a graphical and a numerical approach of stem form and stem volume estimation. The natural form-factor is not just a reduction factor used for the estimation of stem volume, but is a valid criterion for the description of the geometrical shape of the stem. The method was found to be an efficient and accurate means for the study of form and volume.

If Hohenadl's original procedure is applied by using only five sections, a considerable bias is obtained in the lowest fifth (the butt section). Loetsch et al. (1973) reported that this bias can be reduced by measurement of one or several additional diameters in the butt section. Division of the total stem into ten or twenty sections of equal relative length, will evidently also increase the accuracy of the method.

The Modified Hohenadl's Method

Following the suggestion reported by Loetsch et al. (1973), Hoogh et al. (1978) introduced a modification on Hohenadl's original method by dividing the butt section into three additional subsections, measurement of diameters at the midpoint of each one of the subsections and computation of a weighted average, giving a double weight for the diameter measured on the third, closest to the ground subsection. It is believed that an improved and more accurate procedure results.

This modified Hohenadl's method was used for the collection of the data needed for this research study. It was applied to each sample tree and consisted of the following steps:

- measurement of the outside bark $d_{1.30m}$ on the standing tree
- after felling the tree with a chainsaw and observing a standardized 0.15m stump height, the total stem length above stump height was measured and sectioned into five equal length sections (Figure 7). Due to butt swell, the fifth section was subdivided into three additional subsections.
- diameter and bark thickness were measured on the midpoint of the four upper sections and of the three subsections of the butt region. These measurements were done across the upper face as the stem lay on the ground.

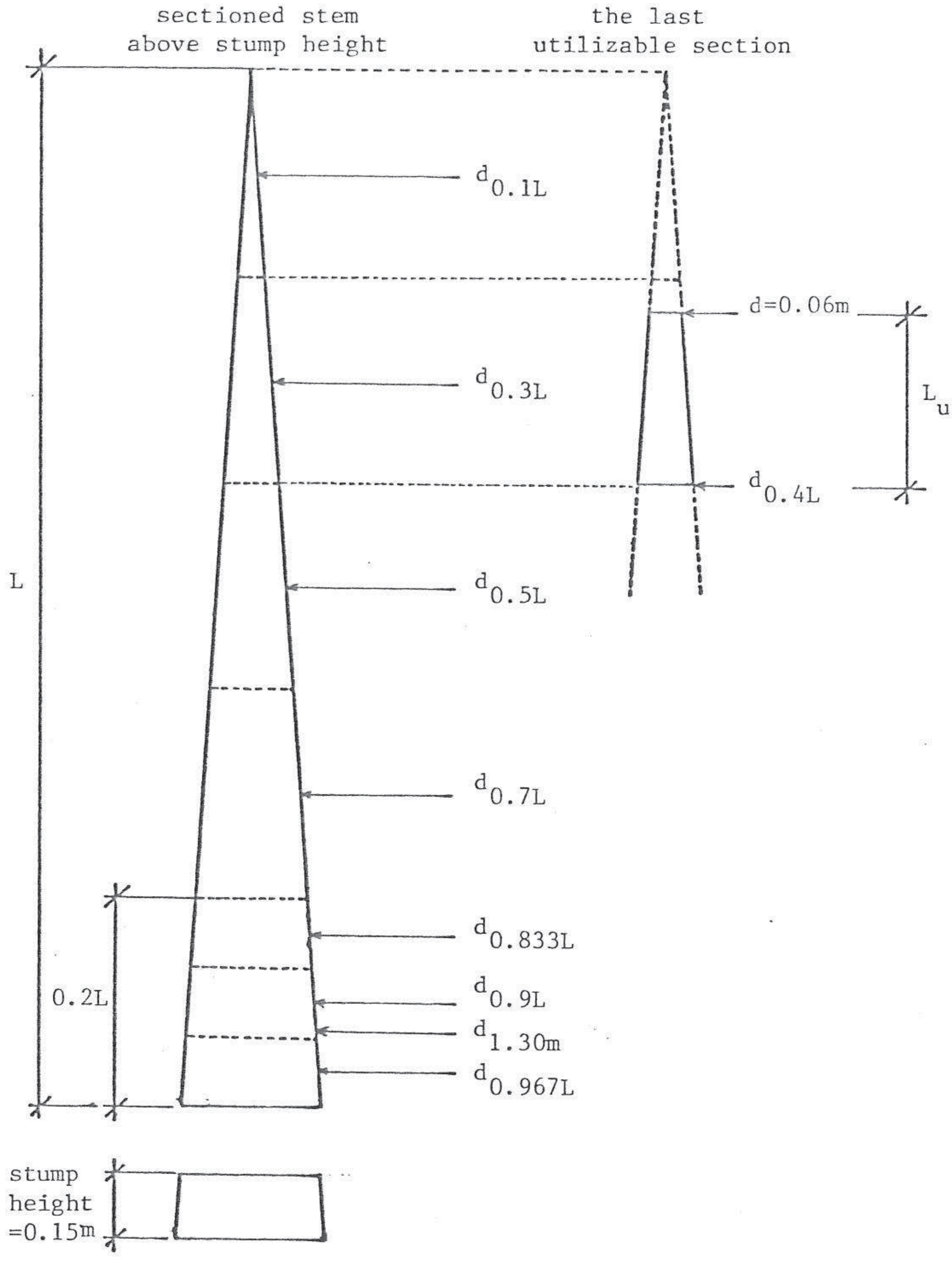

Figure 7. Diameter Measurements Along the Stem According to Modified Hohenadl's Method

- the position of a 6.0cm inside bark diameter limit of utilization was determined on the stem, and the length of the last utilizable section was measured.
- also the diameter inside bark at the base of this last utilizable section was measured and recorded.
- the tree height was determined by adding the stump height to the stem length above stump.

The instruments used for the measurements were an aluminum caliper for the diameters, a Swedish bark gauge for the bark thickness and a steel tape for the determination of the several lengths. All measurements and the computations were done by using the metric system.

An estimate of the volume of interest may be obtained by using the following formulas: (actually only total volume inside bark was computed). Total volume outside or inside bark: (outside or inside bark diameter measurements must be used)

$$V = 0.2L \cdot \frac{\pi}{4} \cdot (d^2_{0.1L} + d^2_{0.3L} + d^2_{0.5L} + d^2_{0.7L}) +$$

$$0.2L \cdot \frac{\pi}{16} \cdot (d^2_{0.833L} + d^2_{0.9L} + 2d^2_{0.967L}) \qquad (B.3)$$

Merchantable volume: since the 6.0 cm merchantable diameter limit is located at a variable position along the stem, the merchantable volume is the sum of the volume of those completely utilized sections and the last partially utilizable portion defined by L_u. For this last section or portion of a section, volume is obtained by Smalian's formula. The following equation exemplifies the procedure for a stem where the first section, beginning from the tip, was not utilized (Figure 7).

$$V = 0.2L \cdot \frac{\pi}{4} \cdot (d^2_{0.5L} + d^2_{0.7L}) +$$

$$0.2L \cdot \frac{\pi}{16} \cdot (d^2_{0.833L} + d^2_{0.9L} + 2.d^2_{0.967L}) +$$

$$L_u \cdot \frac{\pi}{4} \cdot \left(\frac{0.06 + d_{0.4L}}{2}\right)^2 \qquad (B.4)$$

APPENDIX C

FIELD MEASUREMENTS AND VOLUME ESTIMATES

FOR THE SAMPLED LOBLOLLY PINE STEMS

List of Variables Reported and Definitions

TREE = Tree or stem number

D01 . . . D0967 = Diameter inside bark measured at 0.1 . . . 0.967 of stem total length, beginning from the tip (Figure 7). (in meters)

D130 = Diameter at breast height, outside bark, measured at 1.30 m from the ground. (in meters)

HEIGHT = Tree total height (in meters)

TOTVIBHO = Total volume inside bark estimate, according to Hohenadl's method. (cubic meters)

TOTVIBTA = Total volume inside bark estimate, according to the taper-derived volume function. (cubic meters)

RESIDUAL = TOTVIBTA - TOTVIBHO (cubic meters)

$$PCTERR = \frac{RESIDUAL}{TOTVIBHO} \cdot 100 \ (\%)$$

OBS	TREE	D01	D03	D05	D07	D0833	D09	D0957	D130	HEIGHT	TOTVIBHO	TOTVIBTA	RESIDUAL	PCTERR
1	1	0.028	0.102	0.169	0.206	0.230	0.242	0.254	0.278	19.65	0.434868	0.444556	-0.00969	-2.253
2	2	0.030	0.109	0.192	0.315	0.340	0.337	0.377	0.382	21.35	0.916926	0.919735	-0.00281	-0.306
3	3	0.026	0.076	0.126	0.151	0.170	0.190	0.188	0.212	16.95	0.210247	0.211211	-0.00096	-0.459
4	4	0.029	0.086	0.128	0.155	0.176	0.181	0.194	0.211	17.55	0.227943	0.223692	0.00425	1.865
5	5	0.032	0.090	0.119	0.180	0.200	0.220	0.218	0.246	18.65	0.290153	0.318919	-0.02877	-9.914
6	6	0.031	0.092	0.168	0.203	0.220	0.225	0.242	0.262	21.05	0.436294	0.410788	0.02551	5.846
7	7	0.038	0.091	0.155	0.192	0.222	0.229	0.234	0.278	19.25	0.384938	0.422319	-0.03738	-9.711
8	8	0.026	0.064	0.100	0.127	0.148	0.158	0.168	0.187	16.05	0.141687	0.153843	-0.01216	-8.580
9	9	0.034	0.111	0.167	0.195	0.239	0.234	0.256	0.286	19.05	0.417174	0.429640	-0.01247	-2.988
10	11	0.026	0.098	0.163	0.194	0.221	0.228	0.243	0.273	19.15	0.387496	0.428288	-0.04079	-10.527
11	12	0.026	0.048	0.082	0.108	0.132	0.146	0.152	0.175	15.45	0.102393	0.119994	-0.01760	-17.189
12	13	0.029	0.101	0.176	0.236	0.262	0.263	0.294	0.304	18.65	0.509665	0.490370	0.01929	3.786
13	14	0.032	0.092	0.155	0.177	0.203	0.220	0.216	0.256	18.35	0.324794	0.339181	-0.01439	-4.430
14	15	0.021	0.047	0.080	0.110	0.131	0.144	0.162	0.178	15.75	0.107198	0.129540	-0.02234	-20.843
15	16	0.026	0.075	0.115	0.135	0.150	0.161	0.159	0.133	17.55	0.175298	0.171264	0.00403	2.301
16	17	0.031	0.105	0.184	0.230	0.264	0.267	0.284	0.325	19.85	0.539416	0.555243	-0.01583	-2.934
17	18	0.032	0.077	0.104	0.131	0.147	0.152	0.167	0.188	17.05	0.159419	0.150801	0.00862	5.406
18	19	0.036	0.105	0.163	0.208	0.238	0.241	0.265	0.290	20.45	0.465386	0.479055	-0.01367	-2.937
19	20	0.032	0.079	0.128	0.235	0.240	0.249	0.269	0.279	20.45	0.462220	0.487835	-0.02567	-5.553
20	21	0.035	0.098	0.147	0.248	0.273	0.291	0.334	0.345	22.35	0.660897	0.819478	-0.15858	-23.995
21	22	0.031	0.081	0.125	0.151	0.168	0.248	0.230	0.254	19.35	0.293448	0.379867	-0.08642	-29.449
22	23	0.027	0.085	0.132	0.172	0.184	0.189	0.200	0.227	18.35	0.269456	0.263220	0.00624	2.314
23	24	0.022	0.060	0.089	0.112	0.133	0.142	0.148	0.170	17.05	0.119364	0.125432	-0.00607	-5.084
24	25	0.028	0.090	0.127	0.169	0.178	0.188	0.206	0.230	16.75	0.240437	0.233781	0.00666	2.768
25	26	0.027	0.071	0.112	0.135	0.151	0.153	0.163	0.190	17.25	0.164860	0.172699	-0.00784	-4.755
26	27	0.033	0.065	0.090	0.107	0.117	0.125	0.135	0.153	15.55	0.100564	0.094617	0.00595	5.914
27	28	0.023	0.075	0.133	0.160	0.178	0.189	0.206	0.233	18.65	0.254308	0.284571	-0.03026	-11.900
28	29	0.026	0.067	0.091	0.115	0.135	0.131	0.148	0.170	15.25	0.110221	0.114784	-0.00456	-4.139
29	30	0.035	0.089	0.145	0.183	0.198	0.212	0.227	0.242	21.35	0.367847	0.366521	0.00133	0.360
30	31	0.034	0.093	0.168	0.214	0.238	0.245	0.278	0.295	21.75	0.514483	0.556726	-0.04224	-8.211
31	32	0.038	0.083	0.154	0.178	0.195	0.202	0.210	0.234	18.25	0.299924	0.289883	0.01004	3.348
32	33	0.033	0.093	0.138	0.170	0.199	0.205	0.221	0.240	20.85	0.333313	0.350798	-0.01749	-5.246
33	34	0.035	0.074	0.133	0.154	0.159	0.172	0.188	0.205	18.25	0.228345	0.216505	0.01184	5.185
34	35	0.021	0.063	0.103	0.140	0.160	0.169	0.178	0.207	17.05	0.169901	0.183467	-0.01357	-7.985
35	36	0.036	0.104	0.170	0.208	0.235	0.235	0.263	0.294	20.25	0.462459	0.475899	-0.01344	-2.906
36	37	0.032	0.064	0.109	0.140	0.147	0.153	0.160	0.194	18.65	0.184865	0.169594	0.01527	8.261
37	38	0.037	0.100	0.156	0.191	0.208	0.218	0.240	0.265	18.35	0.353591	0.367448	-0.01386	-3.919
38	39	0.031	0.101	0.182	0.244	0.270	0.291	0.315	0.329	19.55	0.595533	0.630476	-0.03494	-5.867
39	40	0.028	0.119	0.172	0.228	0.260	0.263	0.285	0.301	18.65	0.497156	0.502614	-0.00546	-1.098
40	41	0.028	0.094	0.157	0.196	0.218	0.227	0.250	0.272	18.45	0.369950	0.389547	-0.01960	-5.297
41	42	0.028	0.092	0.175	0.194	0.211	0.213	0.254	0.250	19.05	0.396152	0.360226	0.03593	9.069
42	43	0.029	0.090	0.139	0.171	0.197	0.198	0.214	0.230	18.05	0.280124	0.275130	0.00499	1.783
43	44	0.029	0.085	0.138	0.180	0.199	0.205	0.231	0.255	19.25	0.319815	0.350067	-0.03025	-9.459
44	45	0.031	0.123	0.174	0.215	0.222	0.232	0.284	0.284	20.95	0.518501	0.499098	0.01940	3.742
45	46	0.037	0.104	0.161	0.200	0.219	0.229	0.255	0.275	20.75	0.439165	0.446998	-0.00783	-1.784
46	47	0.038	0.111	0.170	0.211	0.233	0.244	0.273	0.280	19.95	0.475567	0.470504	0.00506	1.065
47	48	0.036	0.092	0.173	0.189	0.206	0.214	0.235	0.250	19.35	0.377234	0.362331	0.01490	3.950
48	49	0.030	0.070	0.138	0.174	0.178	0.193	0.201	0.218	16.85	0.242790	0.226314	0.01648	6.786
49	50	0.027	0.066	0.103	0.129	0.148	0.161	0.189	0.184	17.55	0.169872	0.177613	-0.00774	-4.557
50	51	0.029	0.075	0.145	0.162	0.184	0.201	0.210	0.229	17.30	0.254169	0.254238	-0.00007	-0.027
51	52	0.032	0.102	0.161	0.194	0.217	0.225	0.235	0.255	19.60	0.388090	0.383047	0.00504	1.299
52	53	0.031	0.084	0.133	0.148	0.169	0.179	0.195	0.202	18.80	0.239557	0.236794	0.00276	1.153
53	54	0.033	0.087	0.137	0.163	0.179	0.184	0.193	0.205	19.25	0.267304	0.242957	0.02435	9.108
54	55	0.040	0.097	0.150	0.161	0.195	0.204	0.225	0.243	18.85	0.327498	0.306449	0.02105	6.427
55	56	0.038	0.089	0.181	0.193	0.209	0.212	0.223	0.237	19.25	0.379215	0.324467	0.05475	14.437

OBS	TREE	D01	D03	D05	D07	D0833	D09	D0967	D130	HEIGHT	TOTVIBHO	TOTVIETA	RESIDUAL	PCTERR
56	57	0.029	0.093	0.139	0.210	0.237	0.237	0.288	0.289	19.75	0.438805	0.474647	-0.035842	-8.168
57	58	0.030	0.100	0.166	0.200	0.213	0.222	0.240	0.249	19.55	0.398956	0.371229	0.027727	6.950
58	59	0.044	0.137	0.232	0.289	0.309	0.318	0.350	0.360	22.75	0.953002	0.919039	0.033963	3.564
59	60	0.035	0.079	0.148	0.173	0.181	0.189	0.196	0.229	18.55	0.277891	0.249569	0.028322	10.192
60	61	0.034	0.093	0.136	0.158	0.176	0.194	0.207	0.224	19.35	0.276990	0.266463	0.010527	3.801
61	62	0.033	0.086	0.141	0.176	0.199	0.199	0.215	0.254	19.25	0.306788	0.318468	-0.011680	-3.807
62	63	0.034	0.101	0.169	0.197	0.222	0.233	0.252	0.267	20.35	0.432710	0.434108	-0.001398	-0.323
63	64	0.032	0.077	0.122	0.146	0.153	0.166	0.179	0.197	17.45	0.196483	0.194077	0.002407	1.225
64	65	0.030	0.071	0.092	0.164	0.176	0.189	0.217	0.222	17.40	0.220885	0.246330	-0.025442	-11.518
65	66	0.024	0.067	0.095	0.120	0.137	0.144	0.153	0.167	16.75	0.130561	0.134668	-0.004107	-3.146
66	67	0.020	0.084	0.123	0.154	0.155	0.191	0.195	0.207	16.95	0.211933	0.217366	-0.005433	-2.564
67	68	0.039	0.104	0.144	0.175	0.189	0.201	0.203	0.225	17.95	0.288921	0.266613	0.022308	7.721
68	69	0.029	0.077	0.099	0.114	0.128	0.143	0.151	0.169	16.55	0.130971	0.127836	0.003135	2.393
69	70	0.030	0.073	0.097	0.118	0.135	0.144	0.155	0.165	17.15	0.137444	0.134559	0.002884	2.099
70	71	0.044	0.082	0.111	0.127	0.139	0.139	0.154	0.164	17.05	0.155638	0.135055	0.020583	13.225
71	72	0.021	0.043	0.075	0.105	0.129	0.133	0.146	0.144	8.15	0.047979	0.049324	-0.001345	-2.803
72	73	0.027	0.047	0.078	0.112	0.128	0.141	0.164	0.148	9.05	0.061625	0.056464	0.005161	8.375
73	74	0.023	0.042	0.073	0.108	0.129	0.133	0.152	0.144	8.45	0.051395	0.051643	-0.000248	-0.482
74	75	0.024	0.049	0.072	0.104	0.121	0.134	0.150	0.147	9.15	0.054253	0.049139	0.005114	9.427
75	76	0.014	0.040	0.068	0.086	0.094	0.103	0.114	0.111	7.35	0.028473	0.025207	0.003266	11.469
76	77	0.029	0.052	0.071	0.094	0.109	0.122	0.132	0.135	14.45	0.073733	0.072655	0.001075	1.462
77	78	0.029	0.063	0.102	0.149	0.189	0.207	0.223	0.225	12.55	0.161579	0.177906	-0.016327	-10.104
78	79	0.023	0.063	0.113	0.154	0.179	0.179	0.199	0.206	14.05	0.167594	0.179110	-0.011415	-6.807
79	80	0.022	0.050	0.079	0.101	0.117	0.131	0.149	0.143	12.35	0.073280	0.071262	0.002018	2.754
80	81	0.018	0.057	0.091	0.114	0.134	0.138	0.152	0.148	11.75	0.083184	0.078460	0.004724	5.679
81	82	0.020	0.040	0.067	0.073	0.089	0.101	0.112	0.120	11.05	0.038730	0.039461	-0.000731	-1.887
82	83	0.020	0.054	0.085	0.111	0.135	0.142	0.149	0.176	12.15	0.082108	0.102411	-0.020303	-24.727
83	84	0.022	0.062	0.092	0.118	0.136	0.150	0.163	0.159	11.75	0.091561	0.081244	0.010317	11.268
84	85	0.017	0.051	0.078	0.109	0.126	0.141	0.160	0.162	12.15	0.080288	0.085775	-0.005486	-6.833
85	86	0.027	0.062	0.112	0.137	0.160	0.179	0.199	0.201	12.75	0.138736	0.154190	-0.015455	-11.140
86	87	0.018	0.041	0.061	0.080	0.092	0.098	0.100	0.110	10.55	0.034597	0.033584	0.001013	2.927
87	88	0.011	0.039	0.053	0.062	0.070	0.075	0.085	0.081	9.35	0.021010	0.018085	0.002925	13.924
88	89	0.010	0.031	0.050	0.067	0.080	0.080	0.090	0.092	8.35	0.020909	0.020338	0.000571	2.730
89	90	0.013	0.039	0.056	0.070	0.076	0.080	0.084	0.087	9.30	0.023425	0.021021	0.002404	10.262
90	91	0.019	0.041	0.095	0.118	0.137	0.142	0.151	0.157	12.50	0.090203	0.089906	0.000297	0.330
91	92	0.014	0.047	0.070	0.091	0.103	0.132	0.130	0.134	10.75	0.05[illegible]	0.055405	-0.002293	-4.316
92	93	0.017	0.047	0.072	0.094	0.106	0.107	0.122	0.128	10.00	0.045846	0.042418	0.003428	7.477
93	94	0.021	0.063	0.097	0.131	0.159	0.168	0.192	0.195	12.35	0.120326	0.121284	-0.000958	-0.796
94	95	0.016	0.057	0.083	0.124	0.141	0.150	0.161	0.185	12.20	0.093364	0.121961	-0.028596	-30.629
95	96	0.017	0.045	0.066	0.085	0.100	0.105	0.111	0.120	10.55	0.041350	0.041919	-0.000569	-1.376
96	97	0.016	0.035	0.063	0.078	0.099	0.110	0.131	0.113	9.75	0.038588	0.036636	0.001952	5.061
97	98	0.015	0.041	0.067	0.087	0.096	0.113	0.105	0.113	10.75	0.041581	0.037993	0.003588	8.629
98	99	0.023	0.046	0.077	0.097	0.107	0.115	0.134	0.136	10.25	0.052560	0.052614	-0.000054	-0.102
99	100	0.017	0.039	0.056	0.084	0.107	0.113	0.126	0.130	7.15	0.028582	0.030340	-0.001757	-6.149
100	101	0.018	0.029	0.054	0.077	0.094	0.099	0.109	0.102	5.80	0.018291	0.015662	0.002629	14.373
101	102	0.018	0.028	0.051	0.068	0.076	0.082	0.093	0.090	5.15	0.012396	0.012672	-0.000277	-2.233
102	103	0.024	0.031	0.051	0.083	0.093	0.102	0.103	0.098	6.00	0.019255	0.014982	0.004272	22.189
103	104	0.016	0.028	0.052	0.064	0.073	0.079	0.080	0.078	5.20	0.011052	0.010247	0.000805	7.282
104	105	0.020	0.026	0.045	0.058	0.065	0.069	0.073	0.073	4.35	0.008399	0.007930	0.000468	5.577
105	106	0.017	0.038	0.056	0.079	0.098	0.103	0.110	0.114	6.55	0.022331	0.021896	0.000435	1.949
106	107	0.016	0.025	0.051	0.086	0.108	0.113	0.123	0.121	7.13	0.026918	0.029094	-0.002176	-8.085
107	108	0.014	0.022	0.045	0.060	0.052	0.057	0.075	0.065	4.35	0.007390	0.006324	0.001065	14.417
108	109	0.019	0.026	0.057	0.075	0.080	0.083	0.090	0.090	6.25	0.016561	0.015034	0.001526	9.215
109	110	0.015	0.029	0.056	0.079	0.088	0.090	0.097	0.103	6.10	0.017859	0.016794	0.001065	5.965
110	111	0.016	0.024	0.044	0.059	0.064	0.067	0.077	0.065	4.75	0.008208	0.006979	0.001229	14.971

OBS	TREE	D01	D03	D05	D07	D0833	D09	D0967	D130	HEIGHT	TOTVIBHO	TOTVIPTA	RESIDUAL	PCTERR
111	112	0.015	0.027	0.047	0.058	0.064	0.068	0.075	0.059	4.85	0.00850	0.00690	0.00160	18.837
112	113	0.018	0.029	0.042	0.070	0.076	0.080	0.082	0.084	5.09	0.01105	0.01115	-0.00010	-0.910
113	114	0.014	0.032	0.048	0.062	0.069	0.067	0.077	0.069	4.55	0.00874	0.00705	0.00169	19.359
114	115	0.012	0.030	0.058	0.075	0.086	0.091	0.095	0.093	5.45	0.01545	0.01376	0.00169	10.952
115	116	0.018	0.030	0.047	0.061	0.070	0.072	0.075	0.074	5.01	0.00953	0.00855	0.00098	10.283
116	117	0.020	0.031	0.052	0.065	0.071	0.077	0.080	0.086	5.55	0.01207	0.01060	0.00147	12.198
117	118	0.016	0.025	0.045	0.059	0.064	0.068	0.077	0.069	4.35	0.00751	0.00573	0.00158	24.737
118	119	0.020	0.046	0.061	0.084	0.094	0.108	0.112	0.109	5.55	0.02482	0.02373	0.00109	4.395
119	120	0.020	0.030	0.052	0.084	0.112	0.117	0.142	0.132	3.00	0.03416	0.03862	-0.00446	-13.067
120	121	0.018	0.036	0.061	0.084	0.097	0.107	0.116	0.117	7.95	0.02982	0.02904	0.00079	2.634
121	122	0.015	0.038	0.059	0.085	0.083	0.090	0.093	0.099	5.90	0.01847	0.01559	0.00288	15.594
122	123	0.017	0.031	0.057	0.075	0.082	0.098	0.094	0.094	5.25	0.01740	0.01612	0.00128	7.368
123	124	0.018	0.027	0.049	0.067	0.076	0.078	0.083	0.080	4.85	0.01060	0.00875	0.00185	17.444
124	125	0.016	0.028	0.048	0.066	0.059	0.073	0.078	0.077	4.95	0.01000	0.00943	0.00057	5.657
125	126	0.018	0.027	0.047	0.059	0.061	0.067	0.074	0.071	4.45	0.00779	0.00610	0.00169	21.730
126	127	0.018	0.029	0.045	0.060	0.066	0.067	0.071	0.070	4.60	0.00812	0.00598	0.00213	26.270
127	128	0.016	0.023	0.040	0.055	0.060	0.061	0.058	0.054	4.59	0.00666	0.00583	0.00084	12.535
128	129	0.012	0.025	0.043	0.059	0.058	0.063	0.072	0.068	4.75	0.00760	0.00665	0.00095	12.514
129	130	0.017	0.027	0.044	0.058	0.054	0.072	0.078	0.069	5.25	0.00936	0.00789	0.00147	15.743
130	131	0.020	0.036	0.060	0.087	0.102	0.110	0.126	0.129	5.75	0.02740	0.02789	-0.00049	-1.773
131	132	0.043	0.139	0.246	0.294	0.302	0.314	0.327	0.372	26.13	1.09792	1.01115	0.08677	7.903
132	133	0.047	0.147	0.221	0.294	0.311	0.320	0.341	0.380	25.61	1.06786	1.05110	0.01676	1.570
133	134	0.040	0.140	0.200	0.247	0.247	0.259	0.288	0.339	24.70	0.75469	0.77226	-0.01757	-2.328
134	135	0.033	0.104	0.188	0.220	0.232	0.233	0.280	0.335	24.25	0.61281	0.69027	-0.07746	-12.640
135	136	0.046	0.123	0.203	0.239	0.254	0.257	0.275	0.318	24.29	0.70540	0.64471	0.06069	8.604
136	137	0.032	0.119	0.169	0.207	0.227	0.248	0.273	0.317	23.89	0.56726	0.62070	-0.05344	-9.421
137	138	0.060	0.173	0.210	0.255	0.253	0.286	0.327	0.333	20.75	0.76357	0.54599	0.21758	28.495
138	139	0.083	0.152	0.227	0.271	0.288	0.303	0.336	0.392	23.55	0.93766	0.93083	0.00683	0.728
139	140	0.039	0.119	0.196	0.271	0.311	0.293	0.326	0.390	24.63	0.87027	1.01277	-0.14250	-16.374
140	141	0.049	0.124	0.181	0.232	0.251	0.257	0.285	0.332	22.63	0.62585	0.64952	-0.02367	-3.783
141	142	0.042	0.139	0.206	0.269	0.285	0.290	0.299	0.359	22.79	0.78919	0.76474	0.02445	3.098
142	143	0.057	0.127	0.194	0.234	0.255	0.258	0.283	0.341	22.50	0.64847	0.59225	0.05623	8.671
143	144	0.053	0.155	0.229	0.267	0.293	0.298	0.319	0.351	23.41	0.89554	0.85624	0.03931	4.389
144	145	0.051	0.162	0.261	0.318	0.333	0.356	0.406	0.407	24.60	1.30547	1.24029	0.06518	4.993
145	146	0.045	0.154	0.238	0.295	0.312	0.330	0.356	0.390	23.91	1.06155	1.06568	-0.00413	-0.389
146	147	0.056	0.155	0.254	0.307	0.332	0.361	0.367	0.409	25.38	1.24207	1.19724	0.04483	3.610
147	148	0.042	0.142	0.218	0.245	0.253	0.279	0.297	0.331	23.75	0.77494	0.75074	0.02420	3.123
148	149	0.040	0.184	0.271	0.333	0.351	0.369	0.382	0.408	24.27	1.36156	1.22012	0.14144	10.388
149	150	0.055	0.175	0.246	0.277	0.294	0.315	0.334	0.375	24.03	1.02437	0.97766	0.04671	4.560
150	151	0.043	0.157	0.243	0.299	0.347	0.325	0.352	0.382	24.35	1.11534	1.04568	0.06967	6.246
151	152	0.040	0.142	0.228	0.286	0.305	0.315	0.359	0.393	23.95	1.00209	1.10815	-0.10606	-10.584
152	153	0.054	0.153	0.228	0.283	0.323	0.331	0.350	0.390	24.85	1.06052	1.12227	-0.06174	-5.822
153	154	0.053	0.173	0.257	0.311	0.345	0.351	0.386	0.409	25.45	1.32067	1.23815	0.08252	6.248
154	155	0.052	0.149	0.208	0.255	0.265	0.296	0.325	0.372	22.60	0.79405	0.86312	-0.06907	-8.698
155	156	0.059	0.162	0.227	0.298	0.301	0.303	0.319	0.355	23.80	0.99018	0.87523	0.11496	11.610
156	157	0.050	0.158	0.245	0.304	0.329	0.334	0.371	0.396	25.67	1.21733	1.19918	0.01815	1.491
157	158	0.051	0.161	0.243	0.287	0.291	0.299	0.310	0.352	24.85	1.01463	0.89299	0.12164	11.989
158	159	0.060	0.203	0.260	0.310	0.338	0.337	0.386	0.408	25.63	1.36065	1.26006	0.10059	7.393
159	160	0.045	0.132	0.231	0.286	0.318	0.329	0.345	0.338	25.55	1.06313	1.11373	-0.05060	-4.759
160	161	0.048	0.164	0.240	0.275	0.291	0.295	0.339	0.356	23.99	0.98417	0.87819	0.10598	10.769
161	162	0.051	0.169	0.239	0.290	0.316	0.325	0.356	0.385	24.90	1.11688	1.10759	0.00929	0.832
162	163	0.053	0.148	0.256	0.301	0.315	0.315	0.339	0.373	24.15	1.08544	0.95142	0.13403	12.348
163	164	0.052	0.149	0.210	0.265	0.285	0.306	0.324	0.341	24.39	0.89531	0.85523	0.04007	4.476
164	165	0.053	0.159	0.259	0.296	0.310	0.316	0.326	0.357	25.13	1.11796	0.93902	0.17894	16.006
165	166	0.051	0.106	0.199	0.244	0.248	0.268	0.280	0.322	23.90	0.69206	0.66478	0.02727	3.941

OBS	TREE	D01	D03	D05	D07	D0833	D09	D0967	D130	HEIGHT	TOTVI3HD	TOTVIBTA	RESIDUAL	PCTERR
166	167	0.056	0.161	0.256	0.299	0.316	0.322	0.348	0.372	25.17	1.16588	1.04001	0.125873	10.7964
167	168	0.054	0.168	0.234	0.268	0.301	0.310	0.324	0.353	24.35	0.97650	0.90003	0.076472	7.8313
168	169	0.036	0.116	0.192	0.223	0.245	0.249	0.269	0.311	20.35	0.53343	0.53368	-0.000251	-0.0471
169	170	0.043	0.138	0.198	0.231	0.253	0.264	0.278	0.310	23.40	0.67756	0.65470	0.022857	3.3735
170	171	0.035	0.150	0.197	0.234	0.255	0.249	0.290	0.313	23.10	0.68890	0.62833	0.060565	8.7917
171	172	0.040	0.155	0.235	0.281	0.295	0.305	0.334	0.380	24.29	0.98918	0.96923	0.019954	2.0172
172	173	0.056	0.147	0.220	0.273	0.305	0.309	0.327	0.356	23.99	0.93027	0.90218	0.028086	3.0192
173	174	0.052	0.156	0.207	0.224	0.244	0.239	0.247	0.295	25.50	0.71569	0.57500	0.140697	19.6588
174	175	0.053	0.144	0.206	0.249	0.266	0.275	0.308	0.340	23.75	0.78594	0.81095	-0.025015	-3.1829
175	176	0.054	0.147	0.204	0.226	0.255	0.260	0.285	0.336	22.25	0.66399	0.67749	-0.013503	-2.0336
176	177	0.055	0.162	0.204	0.239	0.254	0.246	0.269	0.310	24.25	0.74019	0.63923	0.100959	13.6397
177	178	0.039	0.130	0.203	0.249	0.274	0.271	0.318	0.355	25.25	0.82529	0.89904	-0.073751	-8.9363
178	179	0.044	0.138	0.200	0.242	0.263	0.277	0.308	0.353	23.75	0.75421	0.82504	-0.070831	-9.3914
179	180	0.050	0.133	0.231	0.258	0.274	0.279	0.310	0.343	23.45	0.82859	0.77035	0.058241	7.0289
180	181	0.043	0.139	0.199	0.254	0.252	0.283	0.310	0.322	23.45	0.77049	0.71204	0.058453	7.5864
181	182	0.046	0.146	0.215	0.248	0.259	0.262	0.291	0.313	23.15	0.74942	0.66679	0.082633	11.0263
182	183	0.027	0.103	0.176	0.228	0.255	0.256	0.280	0.301	19.59	0.50732	0.53274	-0.025411	-5.0089
183	184	0.039	0.103	0.174	0.224	0.247	0.258	0.271	0.295	18.55	0.46590	0.46282	0.003076	0.6602
184	185	0.036	0.120	0.173	0.216	0.235	0.247	0.257	0.290	20.19	0.48592	0.50486	-0.018937	-3.8972
185	186	0.037	0.117	0.173	0.226	0.234	0.257	0.272	0.308	20.93	0.53289	0.53961	-0.006722	-1.2614
186	187	0.040	0.119	0.191	0.230	0.251	0.259	0.290	0.293	22.21	0.62274	0.58071	0.042030	6.7493
187	188	0.022	0.056	0.082	0.104	0.129	0.136	0.141	0.151	12.80	0.07925	0.08257	-0.003312	-4.1801
188	189	0.022	0.065	0.103	0.124	0.134	0.141	0.146	0.158	13.48	0.10639	0.09927	0.007123	6.6954

VITA

Sergio Ahrens

Candidate for the Degree of

Master of Science

Thesis: A MATHEMATICAL EXPRESSION OF STEM FORM AND VOLUME OF LOBLOLLY PINE IN SOUTHERN BRAZIL

Major Field: Forest Resources

Biographical:

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